Doll Shivaprasad. H

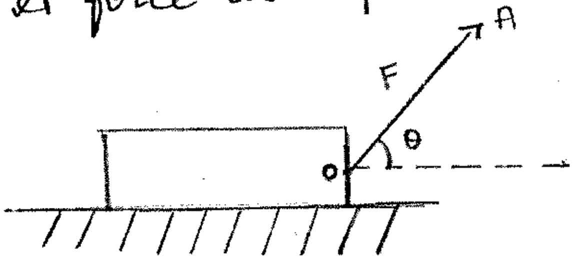
Resultant of Coplanar force System

Force :-

An external agency which changes or tends to change the state of rest or of uniform motion of a body upon which it acts is known as 'force'.

Force is a vector quantity, since it has both magnitude and direction.

A force is represented by four quantities;



* Magnitude 'OA'
* Direction $\overline{OA}$
* Point of application 'O'
* Angle of inclination 'θ'.

One Newton force :-

It is a force required to produce an acceleration of $1 m/s^2$ in a body of mass 1 Kg.

$\boxed{F = ma}$ ∴ Force = Mass × Acceleration

$1N = 1Kg \times 1 m/s^2$

$\therefore N = Kg \cdot m/s^2$.

Weight 'W' of a body of mass 'm' is a force and should be measured in Newtons.

$\boxed{W = mg}$ ∴ Weight of a body = Mass × {Acceleration due to gravity.

$1 Kg \cdot wt = 1 Kg \times 9.81 m/s^2$

$= 9.81 Kg \cdot m/s^2$

$\therefore 1 Kg \cdot wt = 9.81 N$.

One Kg. wt is the force required to move a ma of one Kg with an accleration equal to gravita

-ional acceleration (i.e $g = 9.81 m/s^2$).

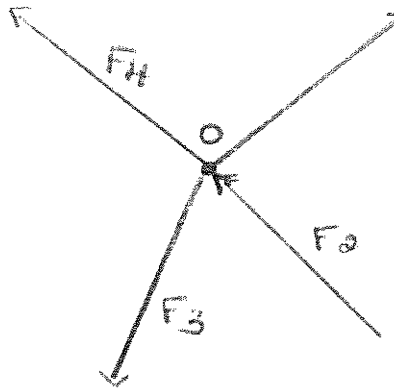
units of force : N, KN, MN, GN.

Classification of force system :-

Coplanar forces :-

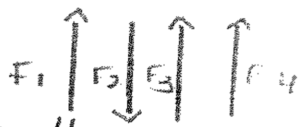
Forces are acting in the same plane.

Coplanar concurrent forces :-



Forces are acting in the same plane and meeting at a point.

Coplanar parallel force :-



Forces are acting in the same plane and are parallel to each other.

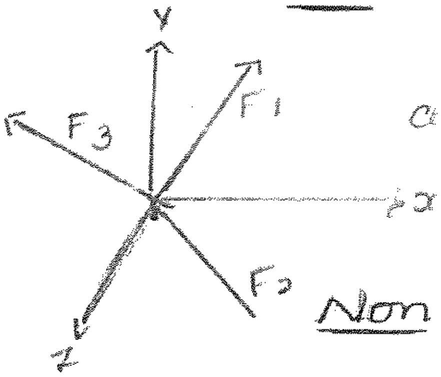
Coplanar non-current forces :-

Forces are acting in the same plane and forces neither meet at a point nor parallel to each other.

Non - coplanar forces :-

Forces are acting in different planes.

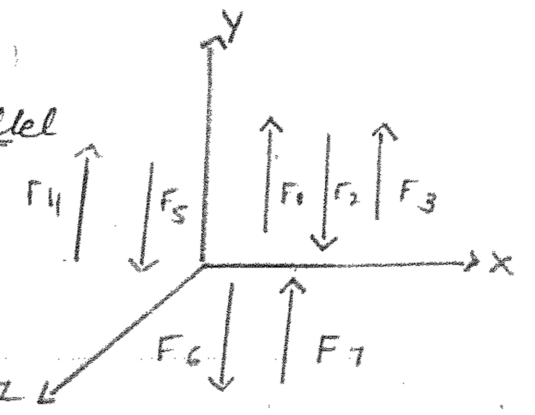
Non-coplanar concurrent forces :-



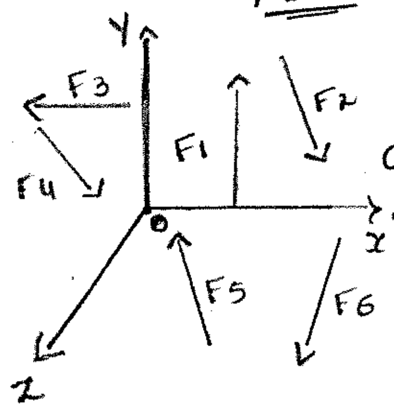
Forces are acting in different planes and meeting at a point.

Non-coplanar parallel forces :-

Forces are acting in a different planes and are parallel to each other.

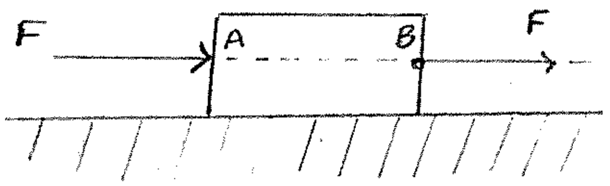


Non-coplanar and Non-concurrent forces:



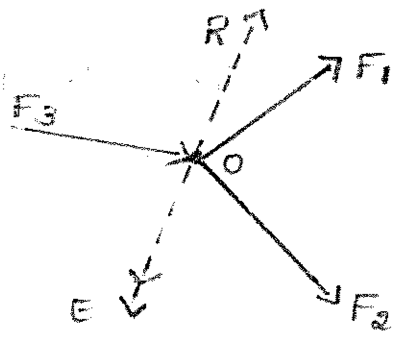
Forces are acting in different planes and forces neither meet at a point nor parallel to each other.

Principle of Transmissibility of force:-



It states that "The point of application of a force may be transmitted (or replaced) to any point along its line of action without changi-ng the conditions of equilibrium"
[i.e, The effect is same].

Resultant [R]:-



If number of forces are acting on a particle, it is possible to find out a single force which will have the same effect as a proceduced by all the forces. This single force is called the Resultant force or Resultant (R).

Equilibrant (E):-

The equilibrant of a system of forces is one which will balance the given system of forces. It is equal in magnitude and opposite in direction to that of the resultant.

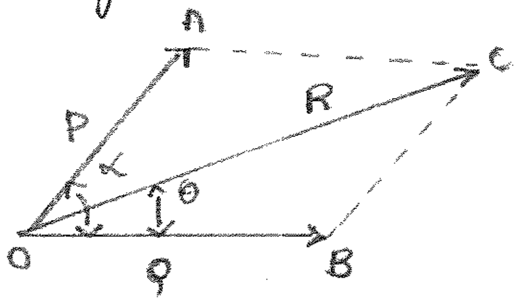
Composition of forces:-

The process of finding the single force [i.e resultant force] which would produce the same effect as that of the given system of forces is called the Composition of forces.

1] Law of parallelogram of forces.
2] Law of triangle of forces.
3] Law of polygon of forces.

1] Law of parallelogram of forces :-

It states that "If two forces, which act at a point be represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from one of its angular points, their resultant (R) is represented by the diagonal of the parallelogram passing through that angular point in magnitude and direction."

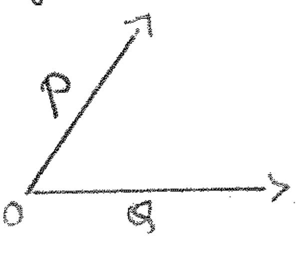


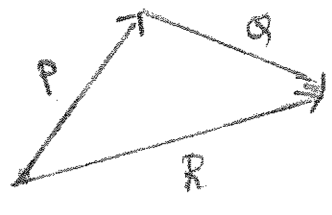
$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$

$$\tan\theta = \frac{P\sin\alpha}{Q + P\cos\alpha}$$

2] Law of triangle of forces :-

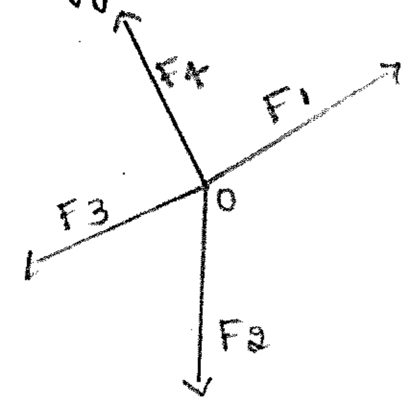
It states that "If two forces acting simultaneously on a particle, be represented in magnitude and direction by the two sides of a triangle taken in order, their resultant (R) may be represented in a magnitude and direction by the third side of the triangle taken in opposite order."

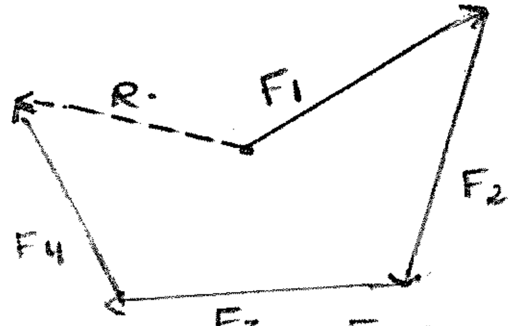




3] Law of polygon of forces :-

It states that "If number of forces acting simultaneously on a particle, be represented in a magnitude and direction by the sides of a polygon taken in order, their resultant (R) may be represent in magnitude & direction by the closing side of the polygon taken in opposite order".



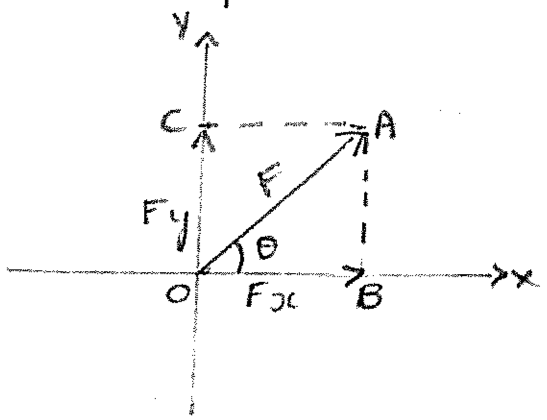


Force polygon

Resolution of a force :-

The splitting up of the force into number of its Components without changing its effect on the body is called resolution of a force.

A force is generally resolved along two mutually perpendicular directions [i.e, rectangul-ar Components].



$$\frac{F_x}{F} = \cos\theta$$

$$\boxed{F_x = F\cos\theta}$$

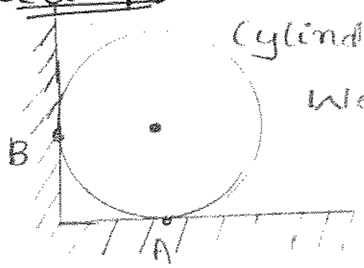
$$\frac{F_y}{F} = \sin\theta$$

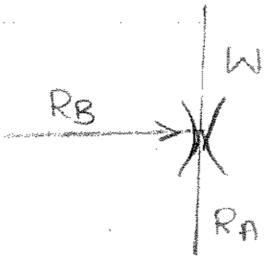
$$\boxed{F_y = F\sin\theta}$$

Free body diagram [FBD] :-

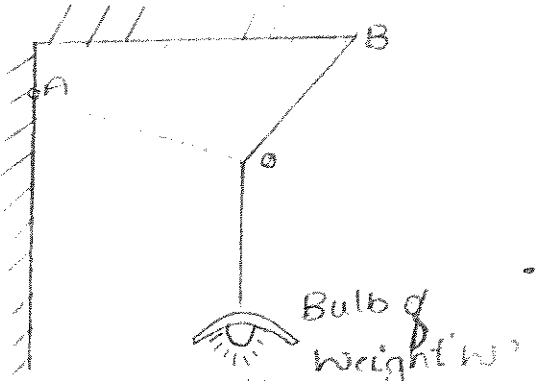
A diagram of the body in which the body under consideration is isolated [or feed] from all the contact surface and all the forces acting on it [including reactions at the contact surface) are shown (or drawn) is called as free body diagram [FBD]

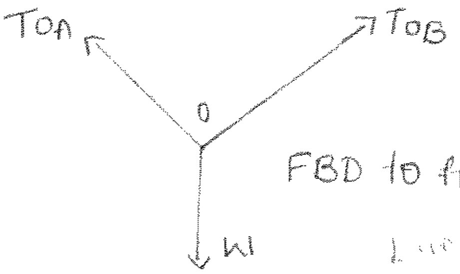
Examples :-



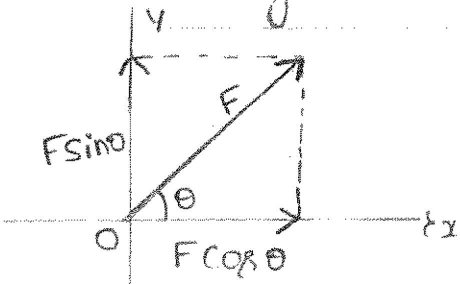


FBD of cylinder to find R_A & R_B

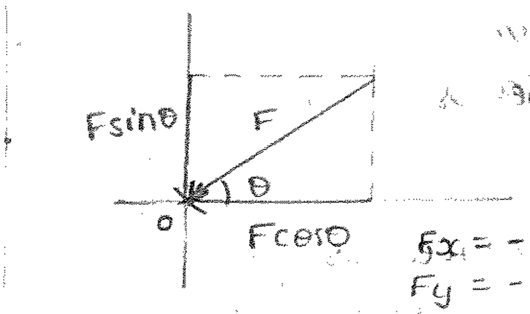




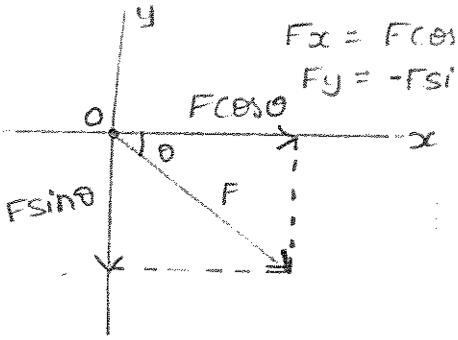
FBD to find T_{OA} & T_{OB}



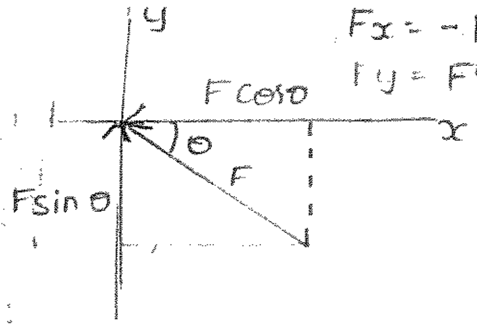
$F_x = F\cos\theta$
$F_y = F\sin\theta$



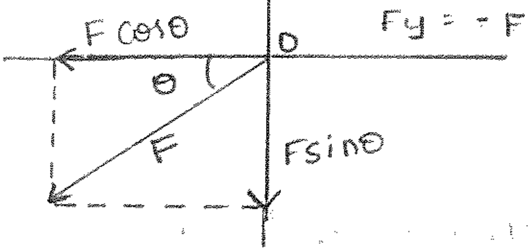
$F_x = -F\cos\theta$
$F_y = -F\sin\theta$



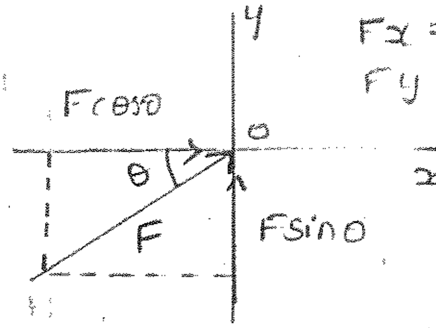
$F_x = F\cos\theta$
$F_y = -F\sin\theta$



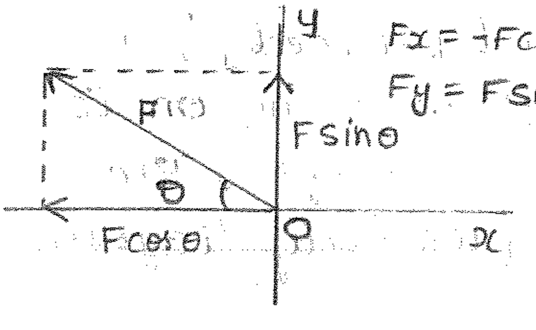
$F_x = -F\cos\theta$
$F_y = F\sin\theta$



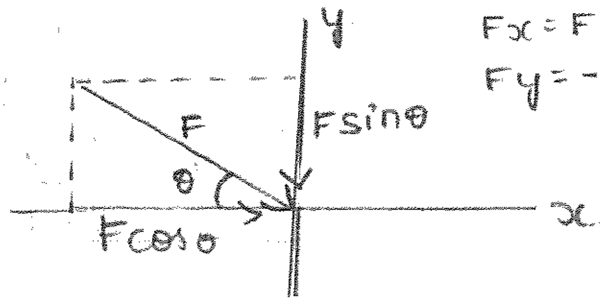
$F_x = -F\cos\theta$
$F_y = -F\sin\theta$



$F_x = F\cos\theta$
$F_y = F\sin\theta$



$F_x = -F\cos\theta$
$F_y = F\sin\theta$

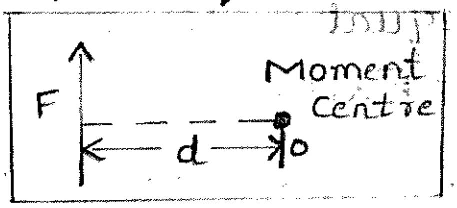


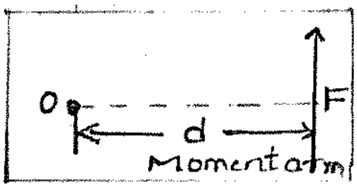
$F_x = F\cos\theta$
$F_y = -F\sin\theta$

Moment of a force :-

Moment of a force about a point is the measure of its rotational effect. The moment of a force is the capacity of a force to rotate the rigid body about any axis.

Moment is defined as the product of the magnitude of force and the perpendicular distance of the point from the line of action of the force.





$\boxed{M = F \times d}$

N-mm, N-m

KN-mm, KN-m

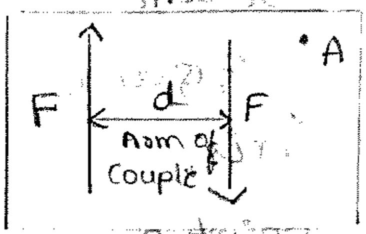
Clockwise moment (+ve)

Anticlockwise moment (-ve)

(Moment is 0 when the line is coinciding of action)

Couple :-

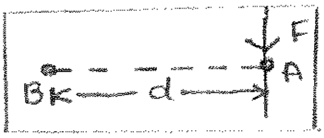
The two parallel forces, equal in magnitude & opposite in direction and seperated by a definite distance are said to form a couple.

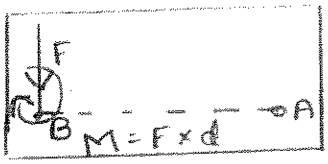


Characteristics of Couple :-

1. The algebraic sum of the components of the two forces is zero. i.e.., The resultant of a couple is zero.
2. The moment of a couple is constant & is equal to the product of one of the forces & $\perp$ distance between them. [$M = F \times d$].
3. The couple can be balanced by equal and opposite couples only.
4. Two or more couples can be reduced to a single couple of moment equal to the algebraic sum of the given couples.
5. The moment of a couple is constant, irrespective of the point such as A, but same for all points in the plane of the couple.

Equivalent force - Couple System :-



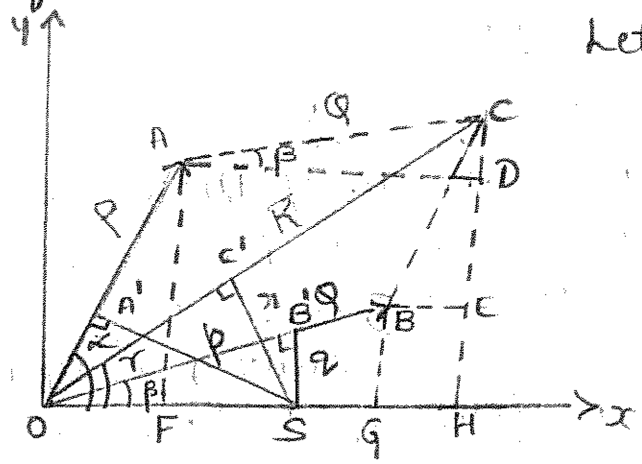


A force acting at any point A on a rigid body may be replaced by another force of the same magnitude and direction at any other point B together with a couple whose moment is equal to the moment of F about the joint B. [i.e., $M = F \times d$]

Varignon's theorem [or principle of moments] :-

The moment of a force [i.e. Resultant] at any point is equal to the algebraic sum of the moments of its components about that point.

Proof :- R is the resultant of P and Q.



Let α, β and γ be the inclinations of P, Q and R w.r to x-axis.

Let S be the point on x-axis.

Draw SA', SB' and SC' ⊥^{r} to the lines of action of forces P, Q and R from S.

Draw AF, BG and CH ⊥^{r} to x-axis and draw AD & BE parallel to x-axis.

From the figure,

$$CH = DH + CD$$

$$\therefore R\sin\gamma = P\sin\alpha + Q\sin\beta$$

Multiplying the above eq^n by the distance OS

$$R(OS)\sin\gamma = P(OS)\sin\alpha + Q(OS)\sin\beta$$

$\therefore R(SC') = P(SA') + Q(SB')$

[But SC' is the moment arm of $R = r$ (say)
SA' — " — " — $P = p$ (say)
SB' — " — " — $Q = q$ (say)

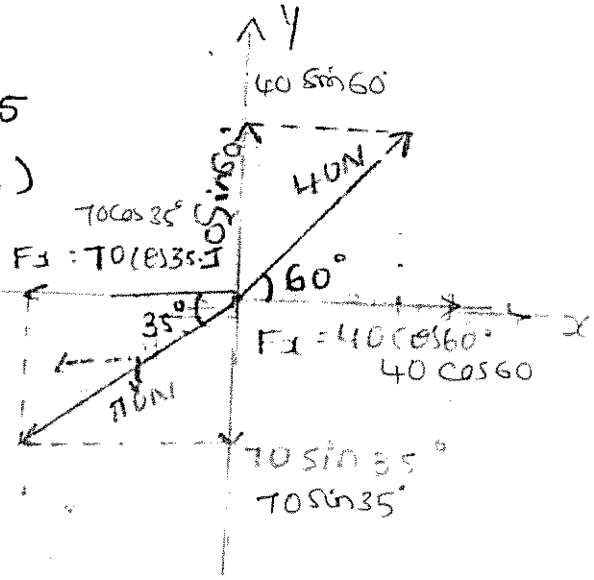
$\therefore \boxed{R.r = P.p + Q.q}$

$\therefore$ Moment of R about S = Algebraic sum of the moments of P and Q about S.

$\therefore$ Moment of the resultant force about a point is equal to the algebraic sum of the moments of its components about that point.

Resultant of Coplanar Concurrent System :-

Find the resultant of Coplanar concurrent force system shown in the figure.



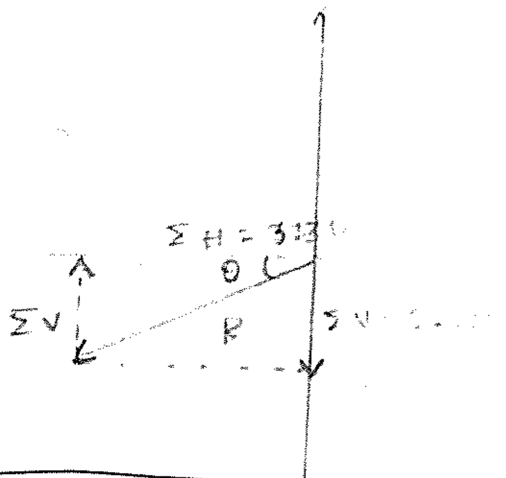
ΣH (Horizontal) $= 40\cos 60 - 70\cos 35$

$= 40 \times \frac{1}{\sqrt{2}} - 70(0.81)$

$= \cancel{40}\ -37.34\,N$

ΣV (vertical) $= 40\sin 60 - 70\sin 35°$

$= -5.5\ N$



$R = \sqrt{(\Sigma H)^2 + (\Sigma V^2)}$

$R = \sqrt{(37.34)^2 + (5.5)^2}$

$R = 37.74\ N$

$\tan\theta = \frac{\Sigma V}{\Sigma H}$

$\theta = \tan^{-1}\frac{(\Sigma V)}{(\Sigma H)}$

$\theta = \tan^{-1}\left(\frac{5.5}{37.34}\right)$

$\boxed{\theta = 8.394°}$

②

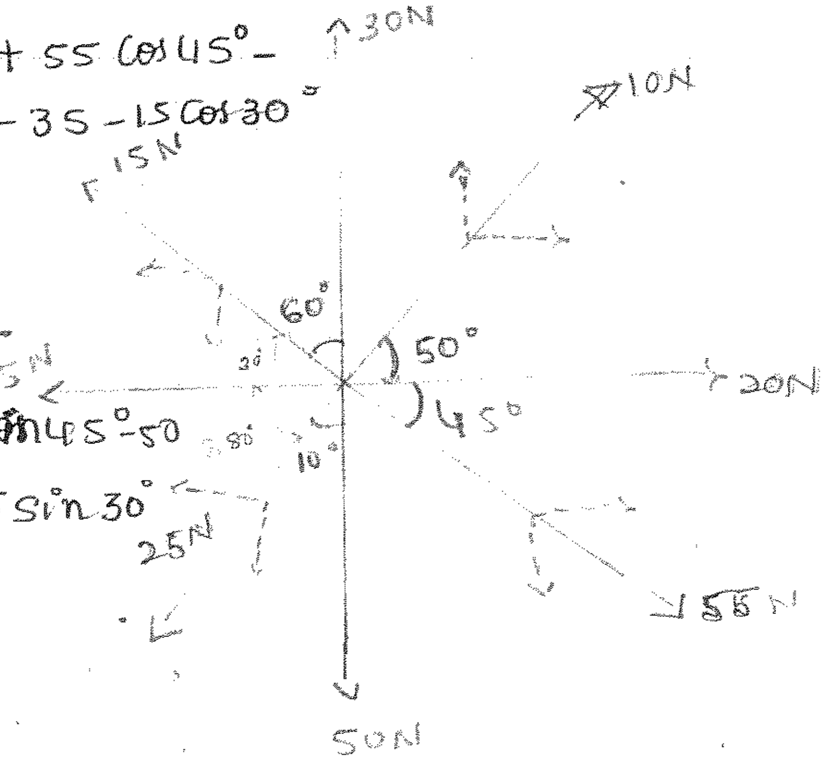
$\Sigma H = 10\cos 50^\circ + 20 + 55\cos 45^\circ - 25\cos 80^\circ - 35 - 15\cos 30^\circ$

~~$\Sigma H = -0.34 N$~~

$\Sigma H = \underline{12.98 N}$

$\Sigma V = 10\sin 50 + 55\sin 45^\circ - 50 - 25\sin 80^\circ + 15\sin 30^\circ + 30$

$\Sigma V = \underline{-68.35 N}$



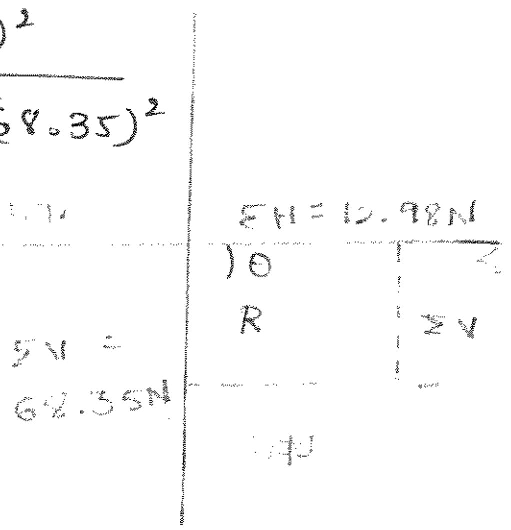
③

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$R = \sqrt{(12.98)^2 + (68.35)^2}$

$R = \sqrt{168.48 + 4671.7}$

$R = \underline{69.57 N}$



$\tan\theta = \dfrac{\Sigma V}{\Sigma H}$

$\theta = \tan^{-1}\left(\dfrac{68.35}{12.98}\right)$

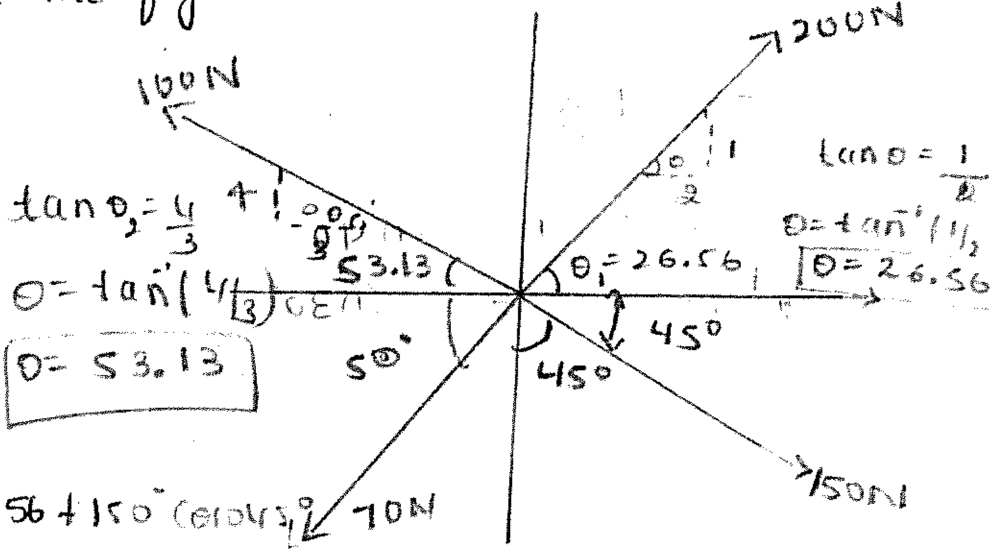
$\theta = \underline{79.24 N}$

$\boxed{\theta = 79.24^\circ}$

④ (i) Explain briefly the classification of force system 5M
July 2023

(ii) State and explain parallelogram law of forces (5M)

(b) A system of four forces acting at a point on a body is as shown in the figure. determine the resultant.

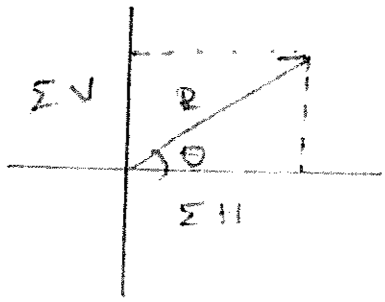


$\Sigma H = 200\cos 26.56 + 150\cos 45° + -70\cos 50° - 100\cos 53.13°$

$\Sigma H = 179.96\ N$

$\Sigma V = 200\sin 26.56 - 150\sin 45° - 70\sin 50° + 100\sin 53.13$

$\boxed{\Sigma V = 9.73\ N}$



$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$R = \sqrt{(179.96)^2 + (9.73)^2}$

$R = \sqrt{32,385.60 + 94.672}$

$\boxed{R = 180.22}$

$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$

$\theta = \tan^{-1}\left(\frac{9.73}{179.96}\right)$

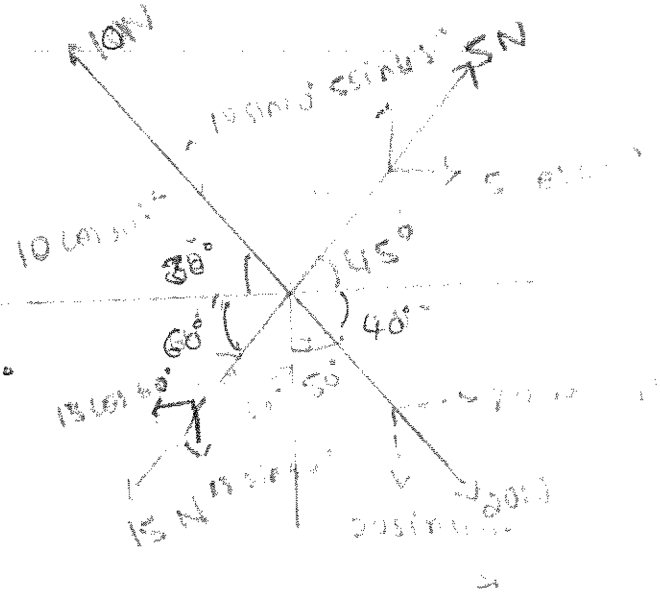
$\boxed{\theta = 3.09°}$

$\Sigma H = -5\cos 45^\circ + 20\cos 40^\circ + 15\cos 60^\circ + 10\cos 30^\circ$

$\Sigma H = -2.69\ N$

$\Sigma V = -5\sin 45^\circ + 20\sin 40^\circ + 15\sin 60^\circ - 10\sin 30^\circ$

$\Sigma V = 17.3\ N$

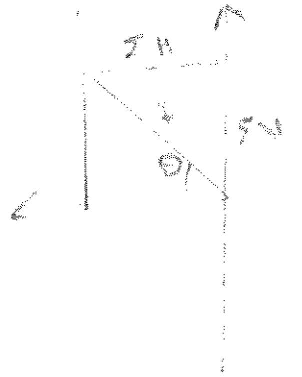


$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$R = \sqrt{(2.69)^2 + (17.3)^2}$

$R = \sqrt{7.236 + 299.29}$

$\boxed{R = 17.5}$

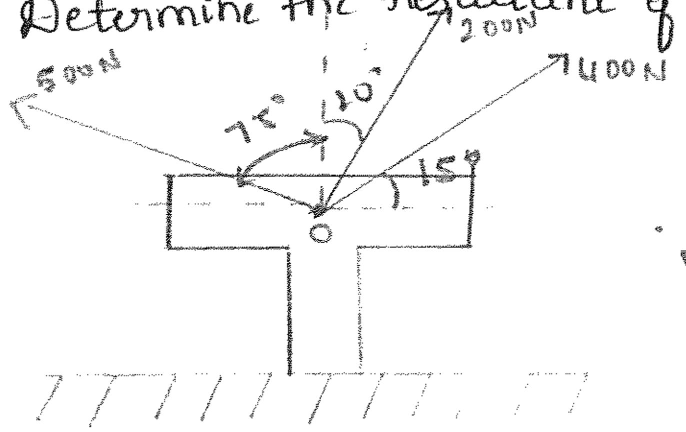


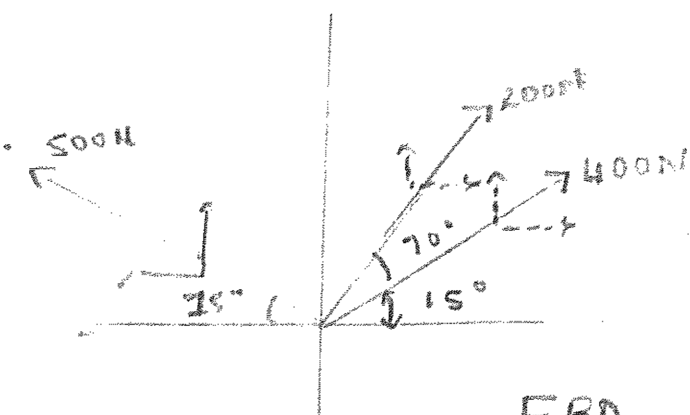
$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$

$\theta = \tan^{-1}\left(\frac{17.3}{2.69}\right)$

$\boxed{\theta = 81.17}$

A bolt is used to anchar
of each wire are shown in the figure.
Determine the resultant of forces acting on it.





FBD

$\Sigma H = 200\cos 70^\circ + 400\cos 15^\circ - 500\cos 15^\circ$

$\Sigma H = -28.18\,N$

$\Sigma V = 400\sin 70 + 400\sin 15 + 500\sin 15^\circ$

$\Sigma V = 420.87\,N$

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

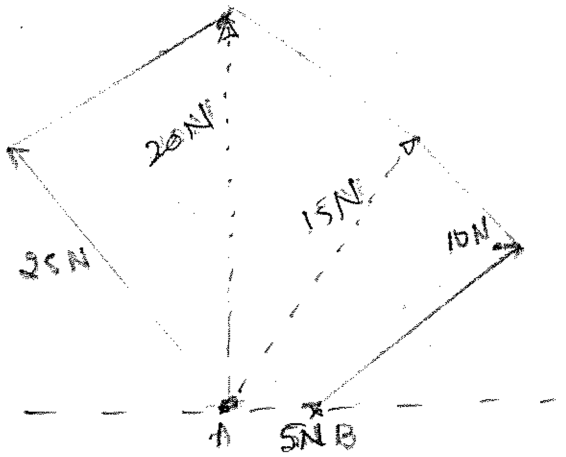
$R = \sqrt{(28.18)^2 + (420.87)^2}$

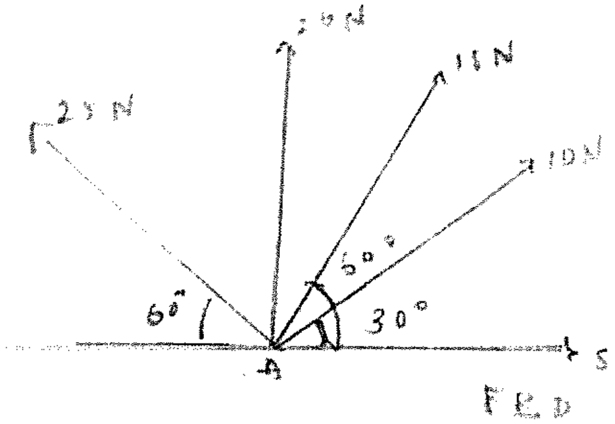
$\boxed{R = 421.81\,N}$

$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$

$\theta = \tan^{-1}\left(\frac{420.87}{28.18}\right)$

$\boxed{\theta = 86.16^\circ}$





$\Sigma H = 15\cos 60^\circ + 10\cos 30^\circ + 5 - 25\cos 60^\circ$

$\Sigma H = 8.6\,N$

$\Sigma V = 15\sin 60^\circ + 10\sin 30^\circ + 25\cos 60^\circ + 20$

$\Sigma V = 59.64$

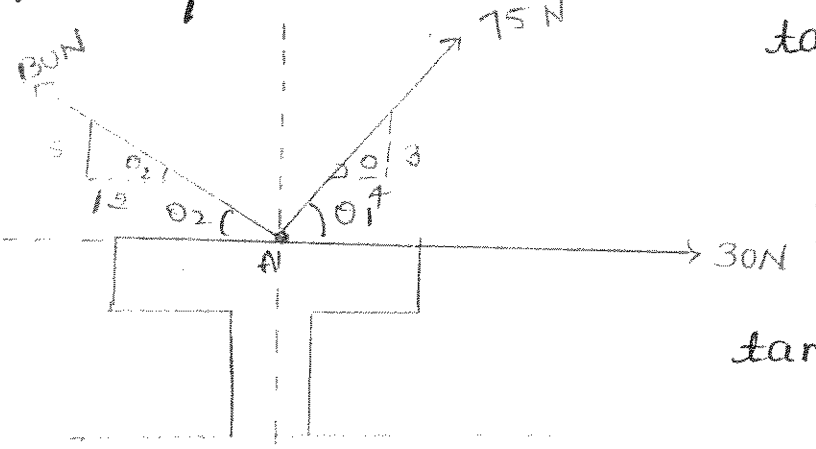
$R = \sqrt{(8.6)^2 + (59.64)^2}$

$\boxed{R = 60.26N}$

$\theta = \tan^{-1}\left(\frac{59.64}{8.6}\right)$

$\boxed{\theta = 81.79^\circ}$

7] A bolt is subjected to 3 forces as shown in the figure. Determine a magnitude and direction of the 4 force required ~~to~~ be applied at A so that the bolt is drawn out with a net resultant vertical force of 325 N.



$\tan\theta_1 = \frac{3}{4}$

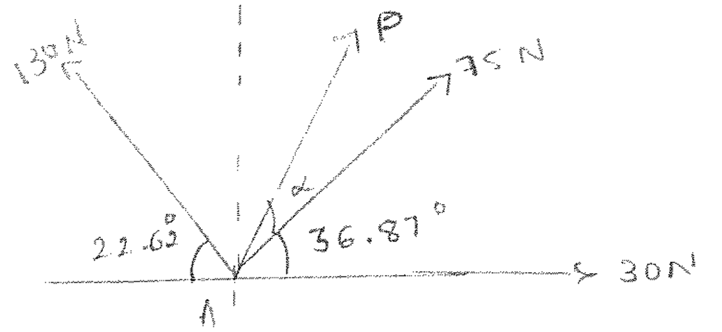
$\theta_1 = \tan^{-1}(3/4)$

$\boxed{\theta_1 = 36.87^\circ}$

$\tan\theta_2 = \frac{5}{12}$

$\theta_2 = \tan^{-1}(5/12)$

$\theta_2 = 22.62^\circ$



FBD

$P = ?\quad \alpha = ?$

$R = 325\text{ N} = \Sigma V$

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$R = \Sigma V$ (given)

$\therefore \Sigma H = 0$

$\Sigma H = P\cos\alpha + 75\cos 36.87^\circ - 130\cos 22.62 = 0$

$P\cos\alpha = 30 \quad \text{—} \textcircled{1}$

$\Sigma V = P\sin\alpha + 75\sin 36.87 + 130\sin 22.62 = 325$

$P\sin\alpha = 230 \quad \text{—} \textcircled{2}$

$\div$ eq ① $\div$ ②

$\frac{\cancel{P}\sin\alpha}{\cancel{P}\cos\alpha} = \frac{230}{30}$

$\tan\alpha = \frac{230}{30}$

$\alpha = \tan^{-1}\left(\frac{230}{30}\right)$

$\boxed{\alpha = 82.5^\circ}$

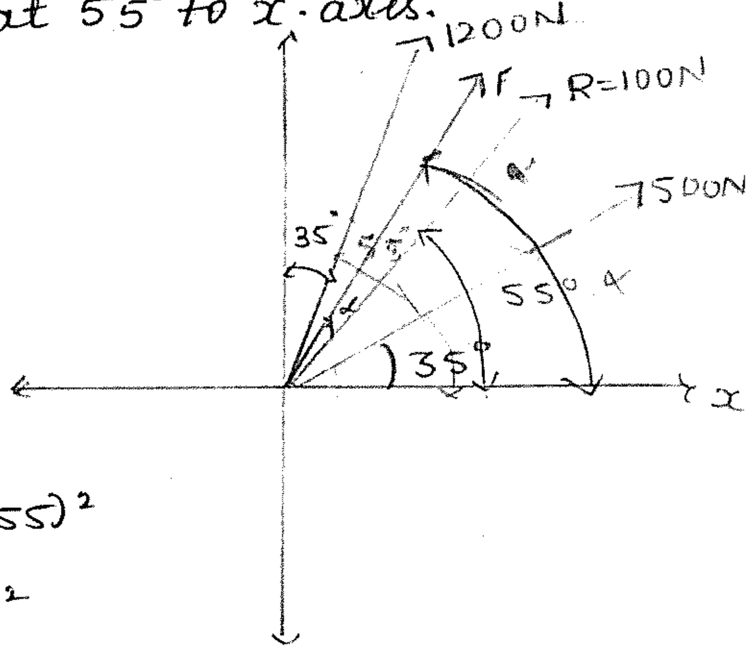
from eqⁿ ①

$P = \frac{30}{\cos(82.5^\circ)}$

$\boxed{P = 232\text{N}}$

June 22

8] Two forces acting on a body are 500N and 1200N as shown in the figure. ~~Become a 3rd fo~~ determine a 3rd force F such that the resultant of all the 3 force is 1000N directed at 55° to x-axis.



$\Sigma H = 1000 \cos 55°$

$\Sigma V = 1000 \sin 55°$

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$R = \sqrt{(1000 \cos 55)^2 + (000 \sin 55)^2}$

$R = \sqrt{(573.5)^2 + (819.15)^2}$

$R = \sqrt{328,902.25 + 671,006.72}$

α = 40.66 F = 691.14N

$\Sigma H = 1200 \cos 55° + 500 \cos 35° + F \cos \cancel{55}° = 1000 \cos 55$

$F \cos \alpha = -524.2 \rightarrow (1)$

$\Sigma V = 1200 \sin 55° + F \sin \alpha \mp 500 \sin 35° = 1000 \sin 55$

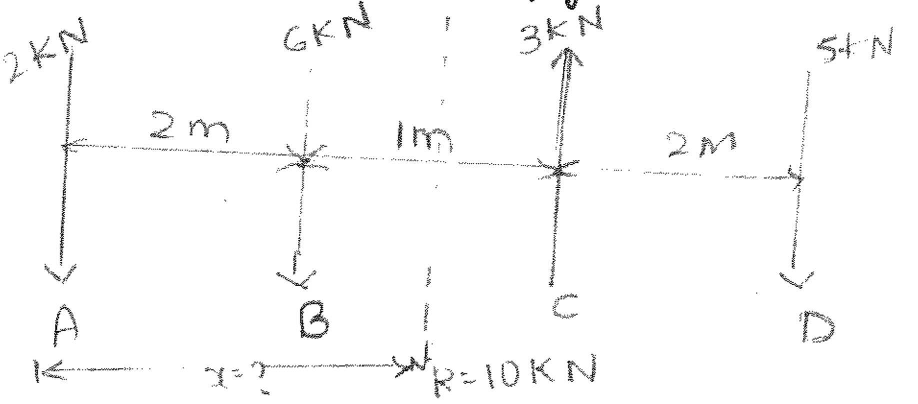
$F \sin \alpha = -450.62 \rightarrow (2)$

$\alpha = \underline{40.68°}$

$F = \dfrac{-524.2}{0.76} = \underline{691.14 \text{ N}}$

Resultant of parallel force system

01] Find the resultant of coplanar parallel force system shown in the figure with respect to point A.



$R = -2-6+3-5$

$R = -10\,KN$

$\therefore \boxed{R = 10\,KN \downarrow}$

$\Sigma M_A = (2\times 0)+(6\times 2)-(3\times 3)+(5\times 5)$

$\Sigma M_A = 28\,KN\text{-}m.$

Using Varignon's theorem

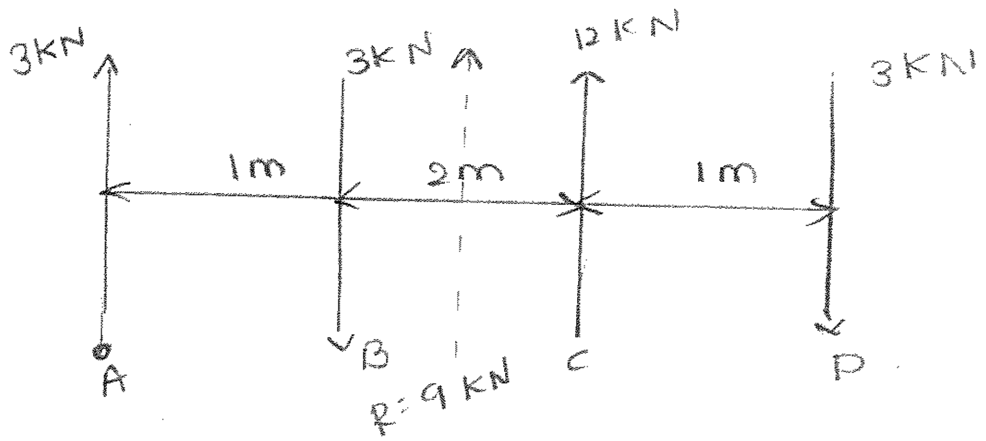
$R \times x = \Sigma M_A$

$10x = 28$

$x = \dfrac{28}{10}$

$\boxed{x = 2.8\,m}$ from A.

02] Find 'R' with respect to A.



$R = 3 - 3 + 12 - 3$

$\boxed{R = 9\ KN} \uparrow$

$\Sigma M_A = (3 \times 0) + (3 \times 1) - (12 \times 3) + (3 \times 4)$

e
nt A.

$\Sigma M_A = 3 - 36 + 12$

$\Sigma M_A = -21\ KN\text{-}m$

Using Varigon's theorem

$R \times x = \Sigma M_A$

$-9x = -21$

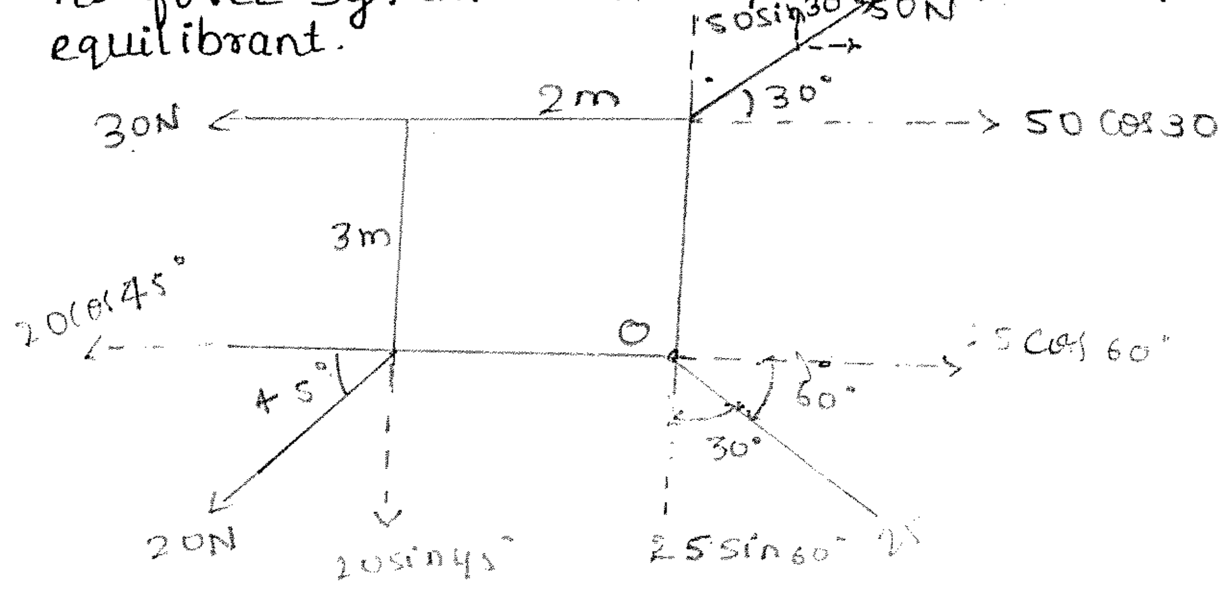
$+x = \dfrac{-21}{-9}$

5x3)

$\boxed{x = +2.3\ m}$

<u>Resultant of Coplanar Non Concurrent force system</u>

01] Find the Resultant of the coplanar non-concurrent force system w.r. to point 'O'. Also find the equilibrant.



$\Sigma H = 50\cos 30 + 25\cos 60 - 20\cos 45 - 30$

$\Sigma H = +11.65\ N$

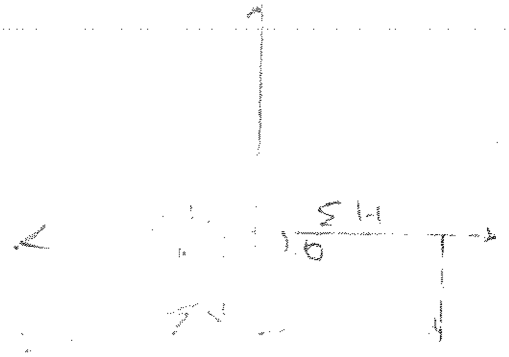
$\Sigma V = 50\sin 30 - 25\cos 60° - 20\sin 45$

$\Sigma V = -10.8\ N$

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$R = \sqrt{(11.65)^2 + (10.8)^2}$$

$$\boxed{R = 15.8\,N}$$



$$\tan\theta = \frac{\Sigma V}{\Sigma H}$$

$$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$$

$$\theta = \tan^{-1}\left(\frac{-10.8}{-11.65}\right)$$

$$\boxed{\theta = 42.83^\circ}$$

$$\Sigma M_O = (50\cos 30 \times 3) - (20\sin 45 \times 2) - (30 \times 3)$$

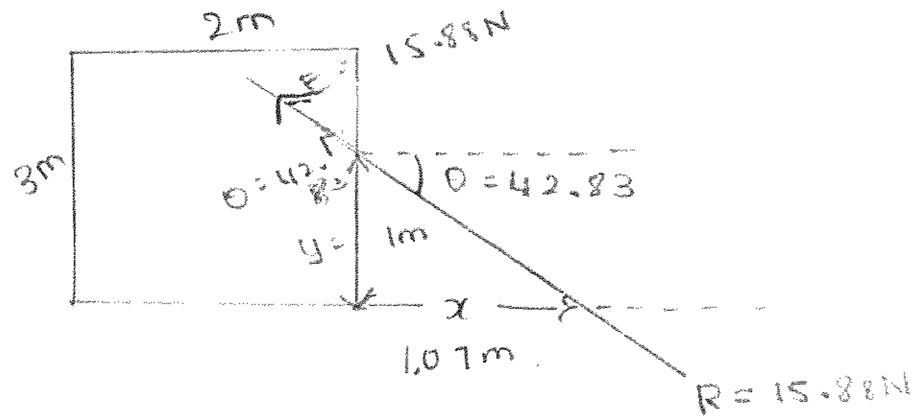
$$\Sigma M_O = 11.62\ N\text{-}m$$

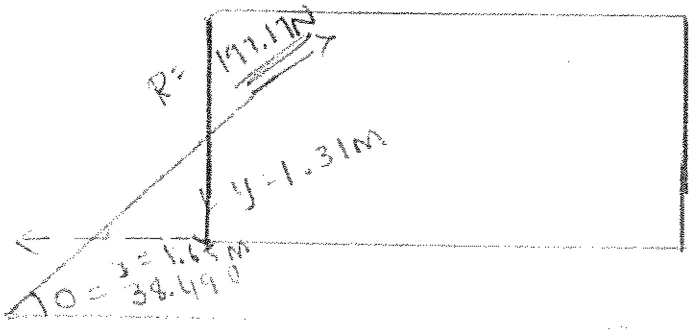
$$x = \frac{\Sigma M_O}{\Sigma V} = \frac{11.62}{10.8}$$

$$\boxed{x = 1.07\,m}$$

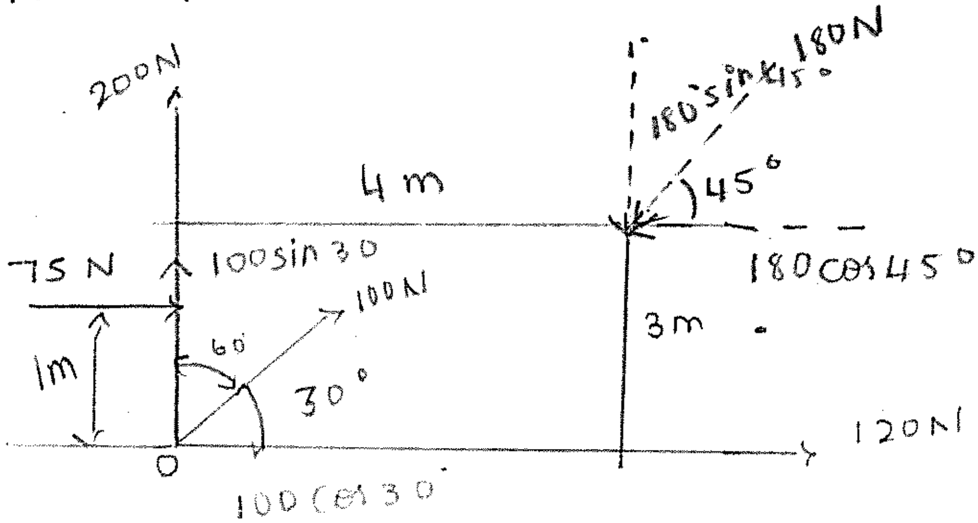
$$y = \frac{\Sigma M_O}{\Sigma H} = \frac{11.62}{11.62}$$

$$\boxed{y = 1\,m}$$



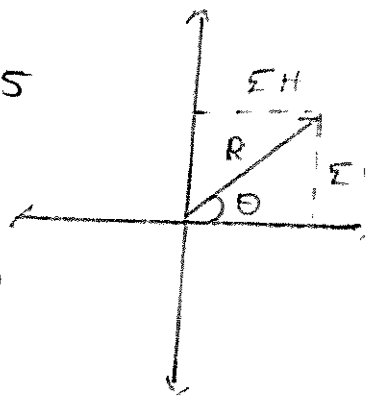


② Find 'R' w.r.to 'O'



$\Sigma H = -180\cos 45^\circ + 120 + 100\cos 30^\circ + 75$

$\boxed{\Sigma H = 154.32N}$



$\Sigma V = -180\sin 45^\circ + 100\cos 30^\circ + 200$

$\boxed{\Sigma V = 122.72N}$

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$R = \sqrt{(154.32)^2 + (122.72)^2}$

$R = \sqrt{23.814.66 + 1560.99}$

$\boxed{R = 197.17N}$

$\tan\theta = \frac{\Sigma V}{\Sigma H}$

$\theta = \tan^{-1}\left(\frac{122.7}{154.3}\right)$

$\boxed{\theta = 38.49^\circ}$

$\Sigma M_O = -(180\cos 45 \times 3) + (180\sin 45 \times 4) + (75\times 1)$
(200×0)

$\Sigma M_O = \underline{202.2\ N\text{-}m}$

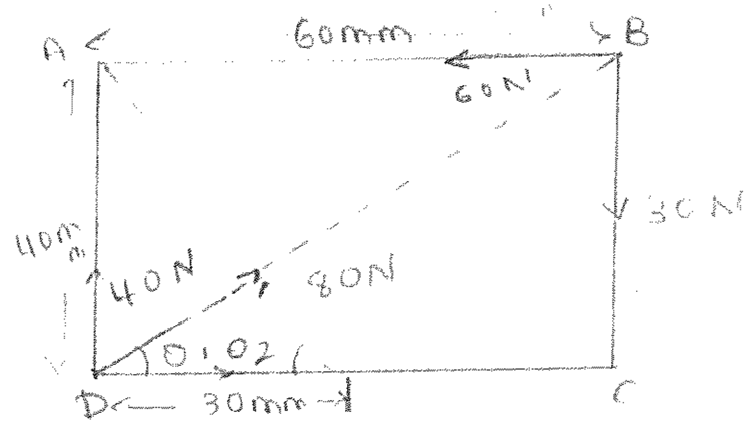
$y = \frac{\Sigma M_O}{\Sigma H} = \frac{202.2}{154.32}$

$\boxed{y = 1.31M}$

$x = \frac{\Sigma M_O}{\Sigma V} = \frac{202.2}{122.72}$

$\boxed{x = 1.65\ m}$

③ Find 'R' w.r.to D.



$\tan\theta_1 = \frac{40}{60}$

$\theta_1 = \tan^{-1}\left(\frac{40}{60}\right)$

$\boxed{\theta_1 = 33.69^\circ}$

$\tan\theta_2 = \frac{40}{30}$

$\theta_2 = \tan^{-1}\left(\frac{40}{30}\right)$

$\boxed{\theta_2 = 53.13^\circ}$

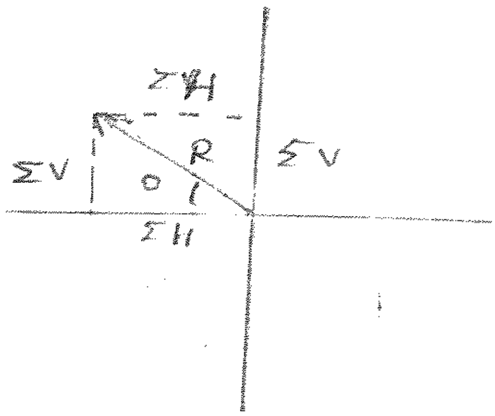
~~ΣH = Cos 33.69~~

$\Sigma H = -60 + 40\cos 33.69 - 80\cos 53.13$

$\Sigma H = -74.72\,N$

$\Sigma V = -30 + 40\sin 33.69 + 80\sin 53.13$

$\Sigma V = 56.19\,N$



$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$R = \sqrt{(74.72)^2 + (56.19)^2}$

$\boxed{R = 93.49N}$

$\tan\theta = \frac{\Sigma V}{\Sigma H}$

$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$

$\boxed{\theta = 36.94^\circ}$

$\Sigma M_D = (30 \times 60) - (60 \times 40) - (80\sin 53.13 \times 30)$

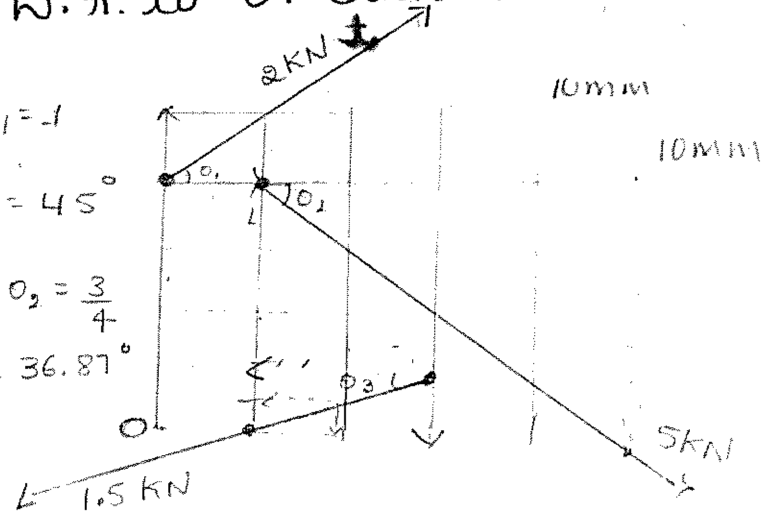
$\Sigma M_D = -2519.9\,N\text{-}m$

$$x = \frac{\Sigma M_D}{\Sigma V} \qquad y = \frac{\Sigma M_D}{\Sigma H}$$

$$x = \frac{-2519.9}{56.19} \qquad y = \frac{-2519.9}{-74.72}$$

$$\boxed{x = 44.84} \qquad \boxed{y = 33.72\ m}$$

Find the resultant w.r.t. to 'O'. Each side is 10mm.



$\tan\theta_1 = \frac{10}{10} \Rightarrow \tan\theta_1 = 1$

$\theta_1 = \tan^{-1}(1) \Rightarrow \theta_1 = 45^\circ$

$\tan\theta_2 = \frac{30}{40} \Rightarrow \tan\theta_2 = \frac{3}{4}$

$\theta_2 = \tan^{-1}(3/4) \quad \theta_2 = 36.87^\circ$

$\tan\theta_3 = \frac{10}{20}$

$\phi_3 = \tan^{-1}(10/20)$

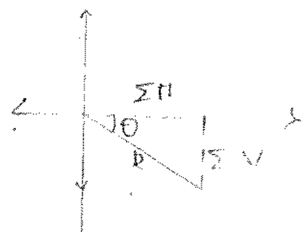
$\theta_3 = 26.56^\circ$

$\Sigma H = 2\cos 45 + 5\cos 36.87 - 1.5\cos 26.56$

$\Sigma H = 4.07\ KN$

$\Sigma V = 2\sin 45 - 5\sin 36.87 - 1.5\sin 26.56$

$\Sigma V = -2.26\ KN.$



$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$R = \sqrt{(4.07)^2 + (2.26)^2}$

$R = 4.65\ KN$

$\tan\theta = \frac{\Sigma V}{\Sigma H} \qquad \theta = \tan^{-1}\left(\frac{2.26}{4.07}\right)$

$\boxed{\theta = 29.04^\circ}$

$\Sigma M_O = (2\cos 45^\circ \times 30) + (5\cos 36.87 \times 30) + (5\cos 1.5 \times 10)$
$\quad - (1.5\cos 26.56 \times 10) + (1.5\sin 26.56 \times 30)$

$\Sigma M_O = 199.13\ KN\text{-}mm$

$x = \frac{\Sigma M_O}{\Sigma V}$ $\qquad$ $y = \frac{\Sigma M_O}{\Sigma H}$

$x = \frac{199.13}{2.26}$ $\qquad$ $y = \frac{199.13}{4.07}$

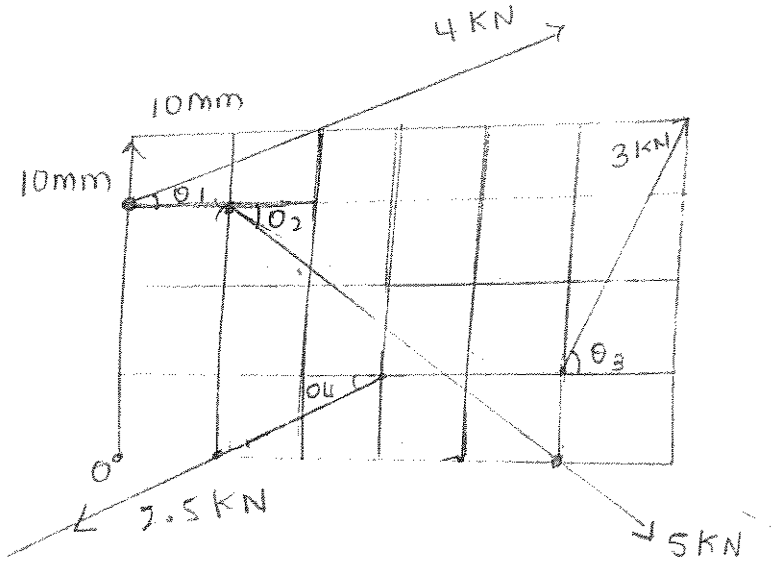
$\boxed{x = 88.11 \text{ mm}}$ $\qquad$ $\boxed{y = 48.93 \text{ mm}}$

July 2023

⇒ State law of transmissibility of forces. [3M|5M]

⇒ State and prove Vareganous theorem or principle of moment [7M]

Find the resultant of a set of coplanar forces w.r to point 'O'. as shown in the figure.
Each side square as 10 mm. [10M]



$\tan\theta_1 = \frac{10}{20}$

$\theta_1 = \tan^{-1}(0.5)$

$\boxed{\theta_1 = 26.56}$

$\tan\theta_2 = \frac{30}{40}$

$\boxed{\theta_2 = 36.86}$

$\tan\theta_3 = \frac{30}{10}$

$\boxed{\theta_3 = 71.56}$

$\tan\theta_4 = \frac{10}{20}$

$\boxed{\theta_4 = 26.56}$

$\Sigma H = 4\cos 26.56 + 5\cos 36.86 + 3\cos 71.56 - 2.5\cos 26.56$

$\Sigma H = 6.29 \text{ KN}$

$\Sigma V = 4\sin 26.56 - 5\sin 36.86 + 3\sin 71.56 - 2.5\sin 26.56$

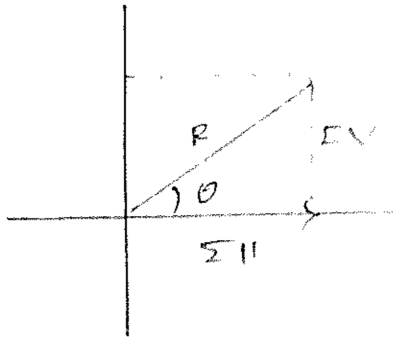
$\Sigma V = 0.51 \text{ KN}$

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$R = \sqrt{(6.29)^2 + (0.51)^2}$$

$$R = \sqrt{39.56 + 0.260}$$

$$\boxed{R = 6.31\ KN}$$



M]
le

$$\tan\theta = \frac{\Sigma V}{\Sigma H} \qquad \theta = \tan^{-1}\left(\frac{0.51}{6.29}\right)$$

$$\boxed{\theta = 4.63^\circ}$$

$$\Sigma M_0 = (4\cos 26.56 \times 30) + (4\sin 26.56 \times 0) +$$
$$(5\cos 36.86 \times 30) + (5\sin 36.87 \times 10) +$$
$$(3\cos 71.56 \times 10) + (3\sin 71.56 \times 30) -$$
$$(2.5\cos 26.56 \times 10) + (2.5\sin 26.56 \times 30)$$

135.66 Nm

$\Sigma M_0 =$

$$x = \frac{\Sigma M_0}{\Sigma V} \qquad y = \frac{\Sigma M_0}{\Sigma H}$$

$$x = \frac{\quad}{0.51} \qquad y = \frac{\quad}{6.29}$$

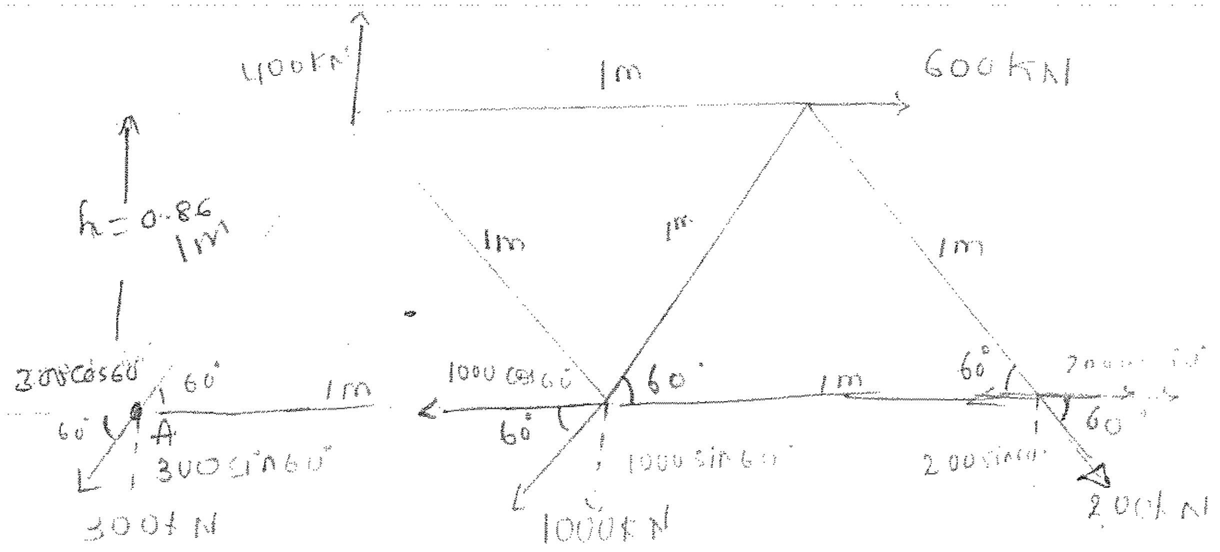
$x = 262.117$

$y = 21.5$

For the system of forces shown in the figure find the resultant w. r to point 'A'

s 26
56

sin
6.56



$$h = \sqrt{1^2 - (0.5)^2} \quad ; \tan 60 = \frac{h}{0.5}$$

$$h = \underline{0.86m} \qquad h = 0.5 \tan 60°$$

$$h = \underline{0.86m}$$

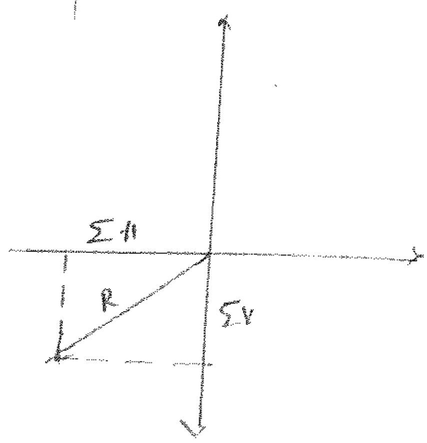
$$\sin 60 = \frac{h}{1} = h = 1 \times \sin 60 \Rightarrow \underline{0.86m}$$

$$\Sigma H = -300\cos 60° - 1000\cos 60° + 200\cos 60° + 600$$

$$\Sigma H = \underline{-150} \text{ KN}$$

$$\Sigma V = -300\sin 60° - 1000\sin 60° - 200\sin 60° + 400$$

$$\Sigma V = \underline{-552.63} \text{ KN}$$



$$R = \sqrt{(\Sigma H)^2 + (\Sigma V^2)}$$

$$R = \sqrt{(150)^2 + (552.63)^2}$$

$$R = \sqrt{22500.0 + 305399.9}$$

$$\boxed{R = 572.62 \text{ KN}}$$

$$\tan\theta = \frac{\Sigma V}{\Sigma H}$$

$$\theta = \tan^{-1}\left(\frac{552.63}{150}\right)$$

$$\boxed{\theta = 74.81^\circ}$$

$$\Sigma M_O = \cancel{300\cos 60^\circ} - (400 \times 0.5) + (600 \times 0.866) + (200 \cos 60^\circ \times 3) - (200 \sin 60 \times 2) + (7000 \sin 60 \times 1)$$

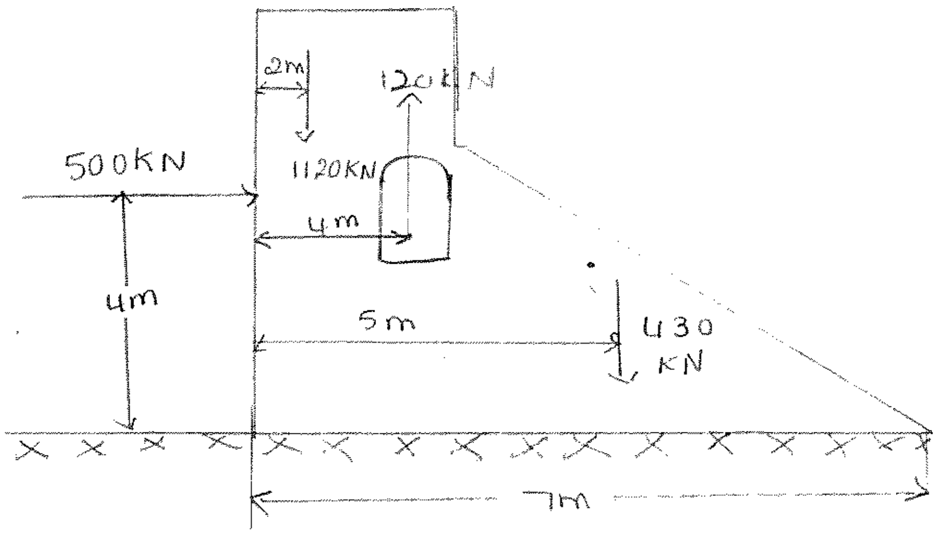
$$\Sigma M_O = \cancel{830.61 \text{ KN mm}} \quad 839.21$$

$$x = \frac{\Sigma M_O}{\Sigma V} \qquad y = \frac{\Sigma M_O}{\Sigma H}$$

$$x = \frac{839.21}{552.63} \qquad y = \frac{839.21}{150}$$

$$\boxed{x = 1.518 \text{ mm}} \qquad \boxed{y = 5.59 \text{ mm}}$$

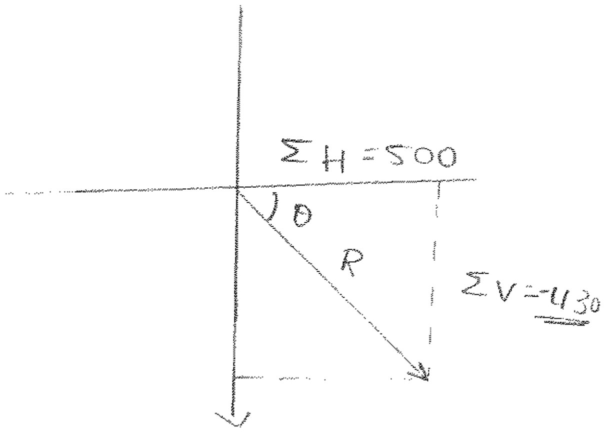
The various forces to be considered for the stability of dam are shown in the figure. The dam is safe if the resultant forces passes through middle ~~thrive~~ third of the base. Verify whether the dam is safe



$\Sigma H = 500\ KN$

$\Sigma V = -1120 + 120 - 430$

$\Sigma V = -1430\ KN$



$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$R = \sqrt{(500)^2 + (1430)^2}$

$R = 1514.89\ KN$

$\tan\theta = \dfrac{\Sigma V}{\Sigma H}$

$\theta = \tan^{-1}\left(\dfrac{1430}{500}\right)$

$\boxed{\theta = 70.72^\circ}$

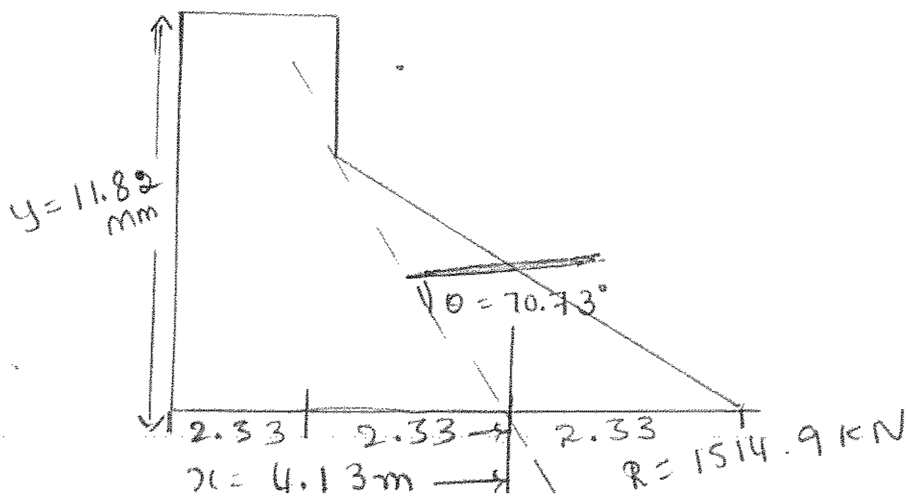
$\Sigma M_O = (500 \times 4) + (1120 \times 2) - (120 \times 4) + (430 \times 5)$

$\Sigma M_O = 5910.$

$x = \dfrac{\Sigma M_O}{\Sigma V}$ $\qquad$ $y = \dfrac{\Sigma M_O}{\Sigma H}$

$x = \dfrac{5910}{1430}$ $\qquad$ $y = \dfrac{5910}{500}$

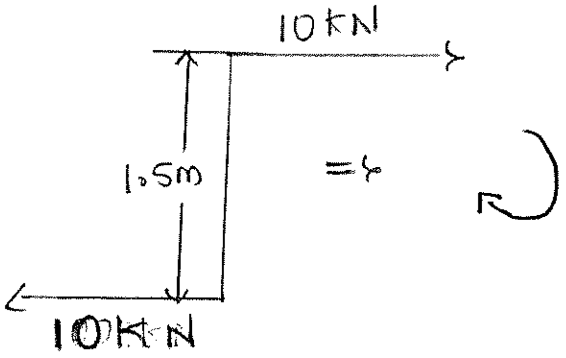
$\boxed{x = 4.13}$ mm from 'O' $\qquad$ $\boxed{y = 11.82}$ mm



∴ The resultant passes through the middle third of the base.

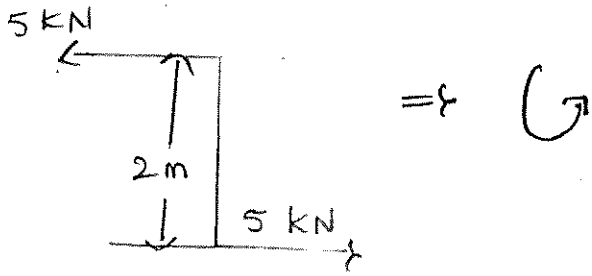
That is between $\frac{1}{3} \times 7 = 2.36$ m & $\frac{2}{3} \times 7 = 4.66$ m from 'O'.

Hence the dam is safe.



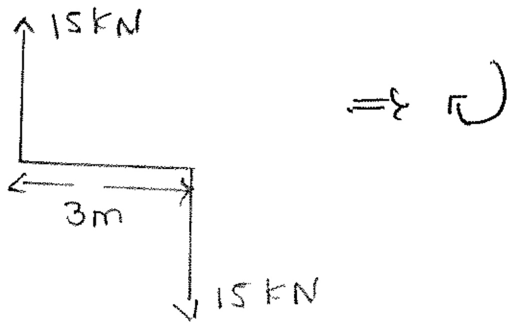
$M = 10 \times 1.5$

$= 15$ KN-m



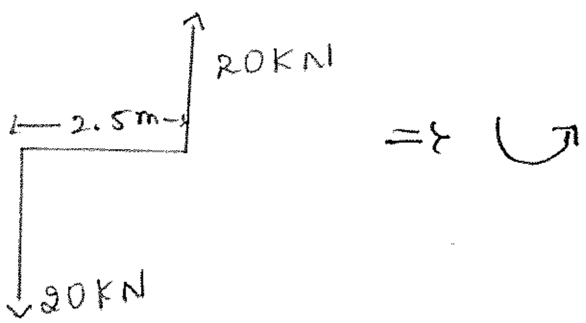
$M = 5 \times 2$

$= -10$ KN-m



$M = 15 \times 3$

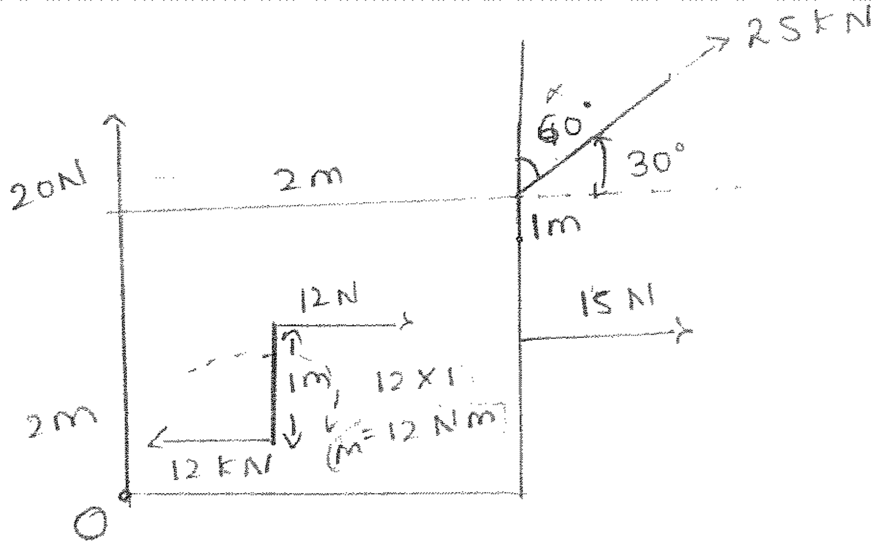
$= 45$ KN-m



$M = 20 \times 2.5$

$M = -50$ KN-m.

8] Find 'R' w.r.to 'O'.



$\Sigma H = 25 \cos 30° + 15 = 36.65\ N$

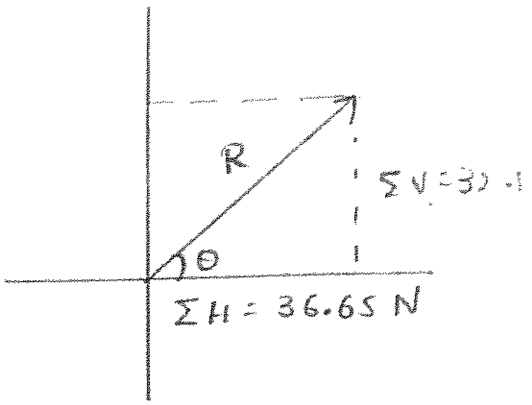
$\Sigma V = 25 \sin 30° + 20 = 32.5\ N$

$R = \sqrt{(\Sigma H)^2 + (32.5)^2}$

$R = \sqrt{(36.65)^2 + (32.5)^2}$

$R = \sqrt{1343.2 + 1{,}056.2}$

$R = 48.98\ \underline{KN}$



$\tan\theta = \dfrac{\Sigma V}{\Sigma H}$

$\theta = \tan^{-1}\left(\dfrac{32.5}{36.65}\right)$

$\boxed{\theta = 41.56°}$

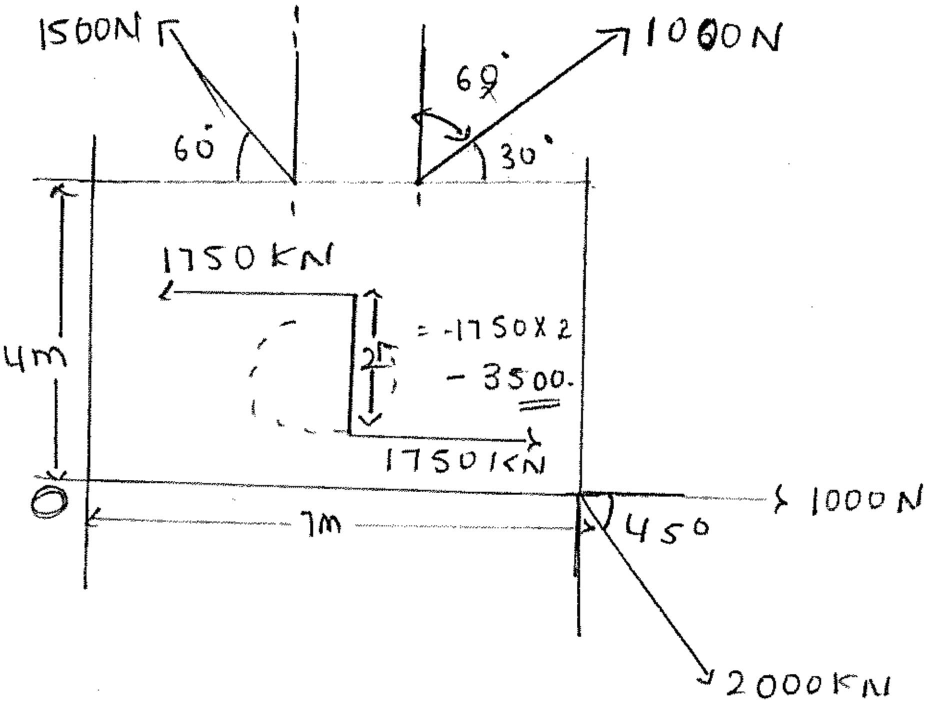
$\Sigma M_O = (20 \times 0) + (25 \cos 30° \times 2) - (25 \sin 30° \times 2) + (15 \times 1) + 12$

$\Sigma M_O = \cancel{33.30}\ 45.30.$

$x = \dfrac{\Sigma M_O}{\Sigma V}$ $\qquad$ $y = \dfrac{\Sigma M_O}{\Sigma H}$

$x = \dfrac{45.30}{32.5}$ $\boxed{x = 1.39\ m}$ $\qquad$ $y = \dfrac{45.30}{36.65}$ $\boxed{y = 1.24\ m}$

Find R w. r to 'O'.

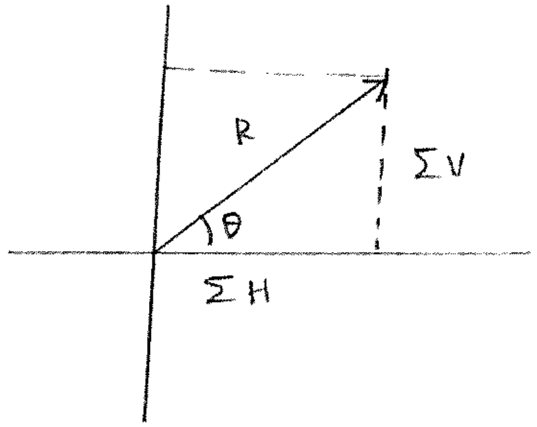


$\Sigma H = 1000\cos 30° + 2000\cos 45° - 1500\cos 60° + 1000$

$\Sigma H = 2,530.24\ N$

$\Sigma V = 1000\sin 30° - 2000\sin 45° + 1500\sin 60°$

$\Sigma V = 384.82\ N$



$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$R = \sqrt{(2530.24)^2 + (384.82)^2}$

$R = \sqrt{6402114.4 + 769.64}$

$\boxed{R = 2559.94\ N}$

$\tan\theta = \left(\frac{\Sigma V}{\Sigma H}\right)$

$\theta = \tan^{-1}\left(\frac{384.82}{2530.24}\right)$

$\boxed{\theta = 8.65°}$

$\Sigma M_O = 1000 \cos 30 \times 4 + (1000 \sin 30 \times 5) + 2000 \sin 45^\circ \times 7) - (1500 \sin 60 \times 3.5) - (1500 \cos 60 \times 4)$

$- 3500$

$\Sigma M_O = -183.30$ N

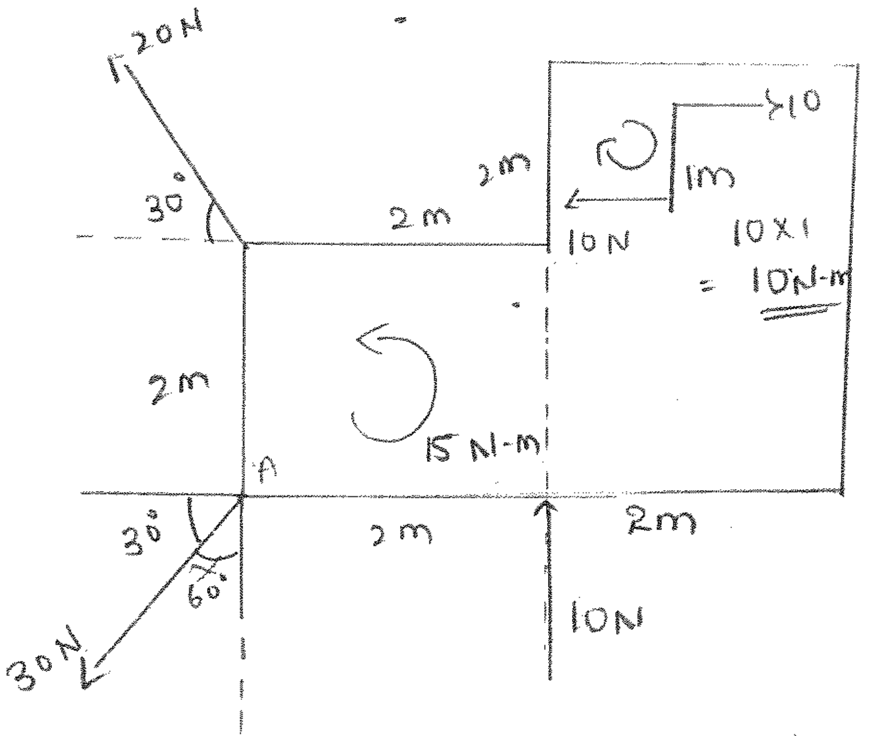
$x = \frac{\Sigma M_O}{\Sigma V}$ $\quad y = \frac{\Sigma M_O}{\Sigma H}$

$y = \frac{183.30}{384.82}$ $\quad y = \frac{-183.30}{2530.24}$

$\boxed{x = 0.47 \text{ m}}$ $\quad \boxed{y = 0.072 \text{ m}}$

April (2021)

Find 'R'. w.r. to. 'A'



M = 10X1

M = 10 N/m

$\Sigma H = -43.3$N

$\Sigma V = 5$N

R = 43.59 N

θ = 6.59°

$\Sigma M_A = -59.64$

$\Sigma H = -20 \cos 30^\circ - 30 \cos 30^\circ$

$\Sigma H = -43.3$ N

$\Sigma V = 20 \sin 30 - 30 \sin 30 + 10$

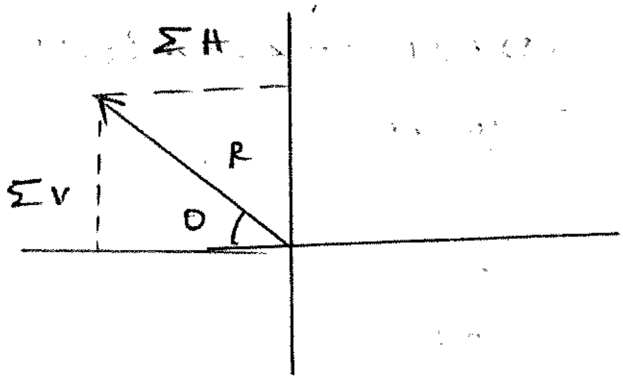
$\Sigma V = 5$N

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$R = \sqrt{(43.3)^2 + (5)^2}$$

$$R = \sqrt{1874.8 + 25}$$

$$\boxed{R = 43.58\ N}$$



$$\tan\theta = \frac{\Sigma V}{\Sigma H}$$

$$\theta = \tan^{-1}\left(\frac{5}{43.58}\right)$$

$$\boxed{\theta = 6.58^{0}}$$

$$\Sigma M_A = (20\cos 30 \times 2) - 10 \times 2 - 15 + 10$$

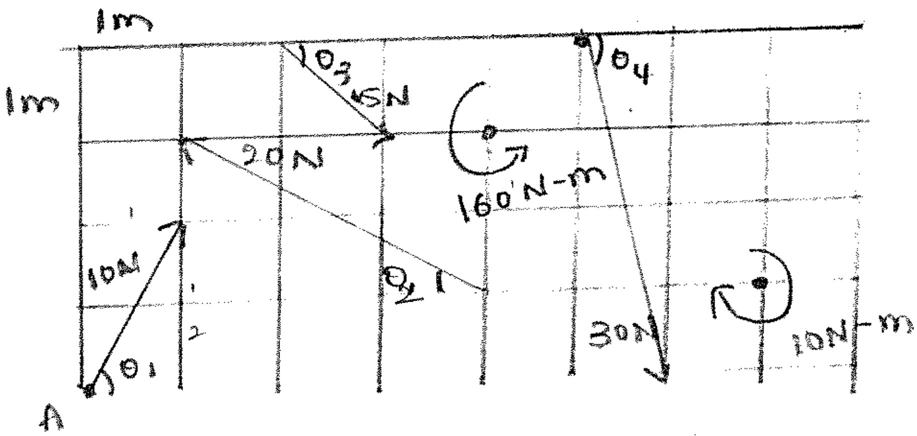
$$\Sigma MA = -59.64\ \underline{N\text{-}m}$$

$$x = \frac{\Sigma MO}{\Sigma V} \qquad y = \frac{\Sigma MO}{\Sigma H}$$

$$x = \frac{+59.64}{5} \qquad y = \frac{59.64}{43.3}$$

$$\boxed{x = 11.92\ m} \qquad \boxed{y = 1.37\ m}$$

(11)



$$\tan\theta_1 = \frac{2}{1} \qquad \tan\theta_2 = \frac{2}{3} = \underline{33.69^{\circ}} \qquad \tan\theta_3 = \frac{1}{1}$$

$$\theta_1 = \tan^{-1}(2/1) \qquad \theta_3 = \tan^{-1}(1/1)$$

$$\tan\theta_4 = \frac{4}{1}$$

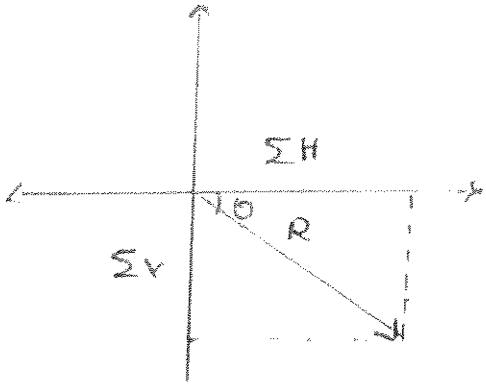
$$\boxed{\theta_1 = 63.43^{\circ}} \qquad \boxed{\theta_4 = 75.96^{\circ}} \qquad \boxed{\theta_3 = 45^{\circ}}$$

$$\Sigma H = 10\cos 63.43 - 20\cos 33.69 + 15\cos 45^\circ + 30\cos 75.96$$

$$\Sigma H = \underline{5.72}\ N$$

$$\Sigma V = 10\sin 63.43 + 20\sin 33.69 - 15\sin 45 - 30\sin 75.96$$

$$\Sigma V = -\underline{19.67}\ N$$



$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$R = \sqrt{(5.72)^2 + (19.67)^2}$$

$$R = \sqrt{32.71 + 386.90}$$

$$\boxed{R = 20.48\ N}$$

$$\tan\theta = \frac{\Sigma V}{\Sigma H}$$

$$\theta = \tan^{-1}\left(\frac{19.67}{5.72}\right)$$

$$\boxed{\theta = 73.78^\circ}$$

$$\Sigma M_A = -(20\cos 33.69 \times 1) - (20\sin 33.69 \times 4) + (15\cos 45 \times 4) + (15\sin 45 \times 2) + (30\cos 75.96 \times 4) + (30\sin 75.96 \times 5) - 160 + 10$$

$$\Sigma M_A = \underline{27.25\ N\text{-}m}$$

$$x = \frac{\Sigma M_A}{\Sigma V} \qquad y = \frac{\Sigma M_A}{\Sigma H}$$

$$x = \frac{27.25}{19.67} \qquad y = \frac{27.25}{5.72}$$

$$\boxed{x = 1.38\ m} \qquad \boxed{y = \cancel{4.17}\ 4.76\ m}$$

15.96

Equilibrium of coplanar force system

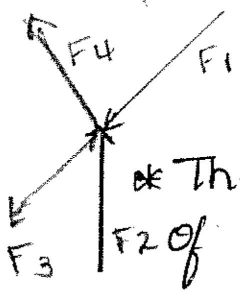
Equilibrium of rigid body

If a rigid body is acted upon by a system of forces and remains at rest, the system is said to be in static equilibrium.

Equilibrium of a rigid body is a state of balance

Equilibrium of coplanar concurrent force system:-

The conditions of static equilibrium to be satisfied are:-



* The algebraic sum of the horizontal components of forces acting on the body is zero. i.e

$$\Sigma H = 0$$

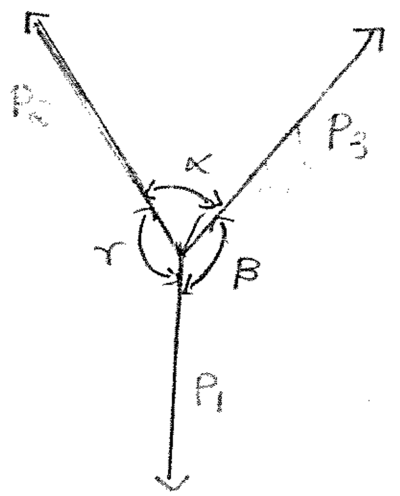
* The algebraic sum of the vertical components of forces acting on the body is zero. i.e,

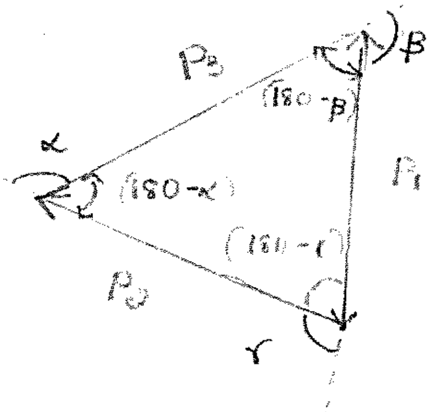
$$\Sigma V = 0.$$

LAMI'S THEOREM:-

96 X4]

It states that "If three forces acting at a point are in equilibrium, each force is proportional to the sine of angle between other two forces."





Force polygon

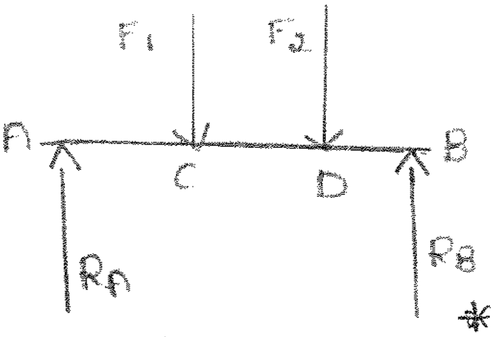
From sine rule,

$$\frac{P_1}{\sin(180-\alpha)} = \frac{P_2}{\sin(180-\beta)} = \frac{P_3}{\sin(180-\gamma)}$$

$$\therefore \boxed{\frac{P_1}{\sin\alpha} = \frac{P_2}{\sin\beta} = \frac{P_3}{\sin\gamma}}$$

Equilibrium of Coplanar parallel force System :-

The conditions of static equilibrium to be satisfied are :-



* The algebraic sum of the Vertical Components of forces acting on the body is zero i.e, $\Sigma V = 0$.

* The algebraic sum of the moments of all forces about any point is zero i.e $\Sigma M = 0$.

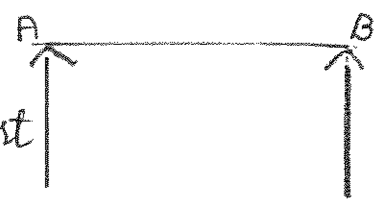
BEAM :-

** A beam is any structural member which carries forces or loads at right angles to the longitudional axis of the member.

Types of Beams :-

1] Simply supported beam :-

A beam supported or resting freely on the walls or columns at its both ends is known as simply supported beam.

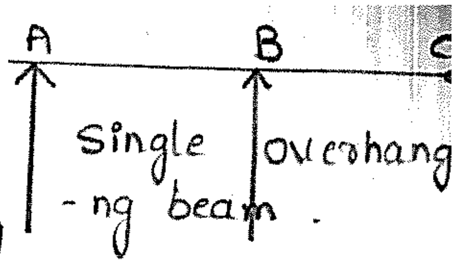


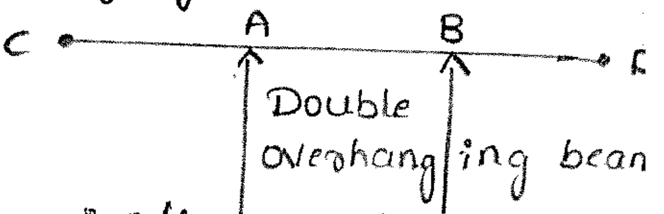
2] Cantilever beam :-

A beam fixed at one end and free at other end is known as Cantilever beam

3] Overhanging beam :-

A beam having its end portions extended beyond the support is known as overhanging beam.



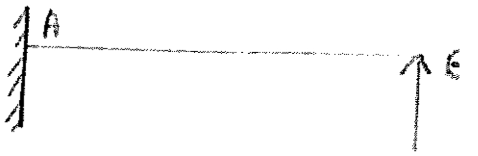


4] Fixed beam :-

A beam whose ends are rigidly fixed @ built in walls is known as fixed beam.

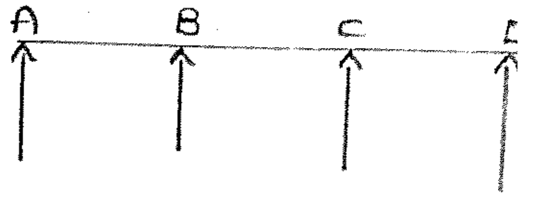
5] Propped cantilever :-

A beam whose is fixed at one end & other end is freely supported on walls @ columns is known as propped cantilever.



6] Continuous beam :-

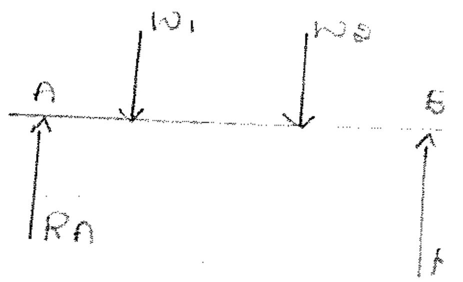
A beam supported on more than two supports is known as continoauses beam.



Types of loadings :-

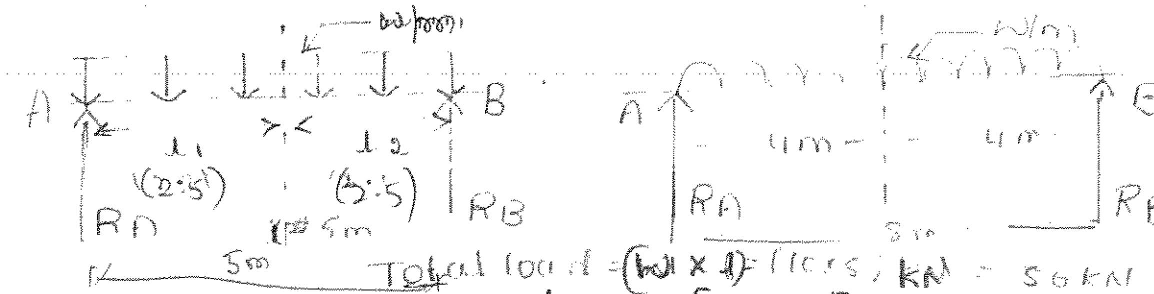
1] Concentrate load @ point load :-

A load acting at a point on a beam is known as Concentrated load @ point load.



2] Uniformly distributed load [UDL] :-

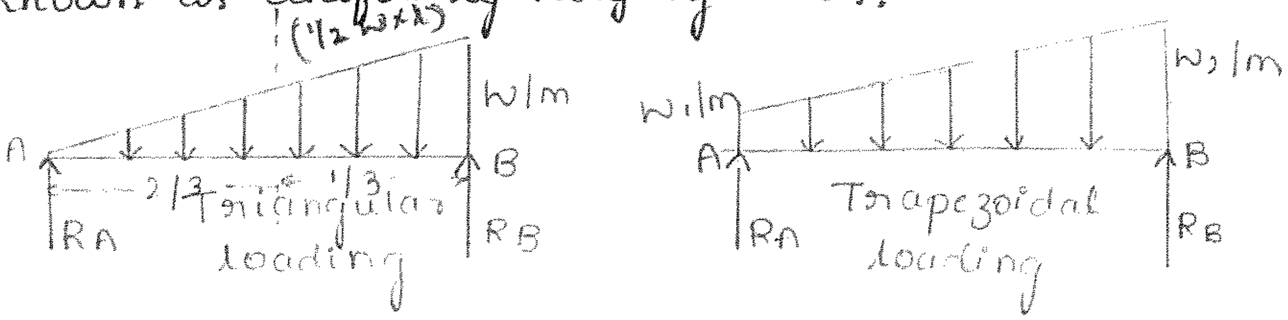
A load which is spred over a beam in such a manner that each unit length is loaded to the same intensity is known as uniformly distributed load.



Total Load = (w×l)

3] Uniformly Varying Load [UVL] :-

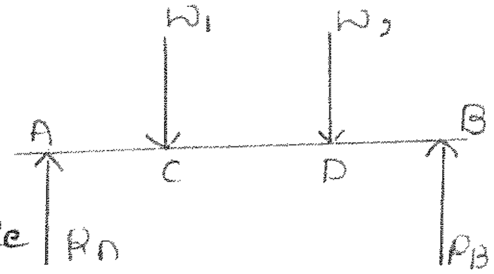
A load is spread over a beam in such a manner that it intensity increases [or Varies] Linearly on each unit length is Known as uniformly varying loads.



Types of Supports :-

1] Simple support :-

The end of the beam rests simply on a rigid support.



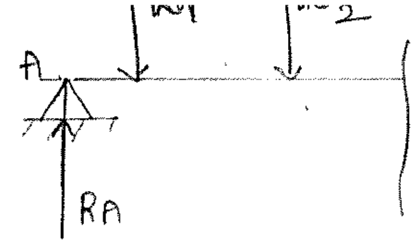
There is no resistance to the force in the direction of the support.

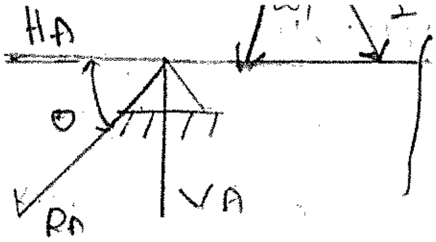
Hence the reaction is always normal to the support

There is no moment resistance at the support.

2] Hinged Support [pinned support] :-

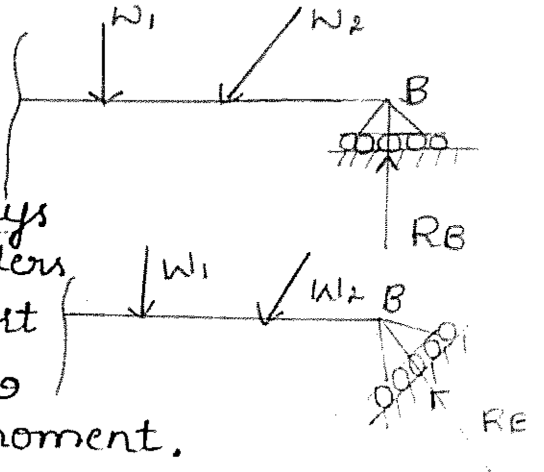
In hinged support, the reaction can be in any direction which is usually represented by its components in two mutually ⊥r direction. This type of support does not provide any resistance to the moment, in other words it permits ritation freely at the end. It is free to rotate.





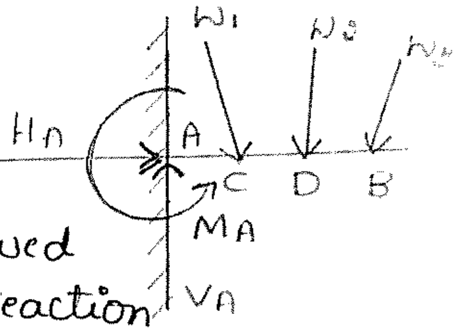
$\sqrt{H_A^2 + V_A^2}$

3] Roller Support :-



In roller support, beam end is supported on rollers. In such cases, reaction is always normal to the support, since rollers are free to roll along the support. The ends are free to rotate also. Hence there is no resistance to moment.

4] Fixed Support :-

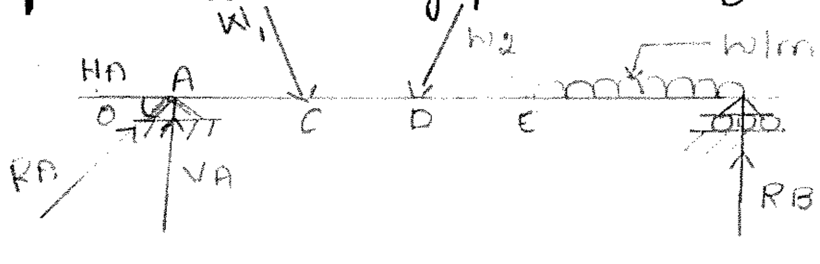


At fixed support the end of the beam is neither permitted to move in any direction nor allowed to rotate. In this support, the reaction [HA & VA) and moment [MA] exists.

Equilibrium of Coplanar non-Concurrent force system :-

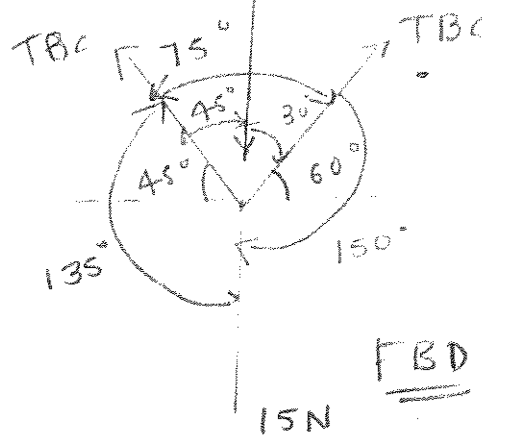
The conditions of static equilibrium to be satisfied are :

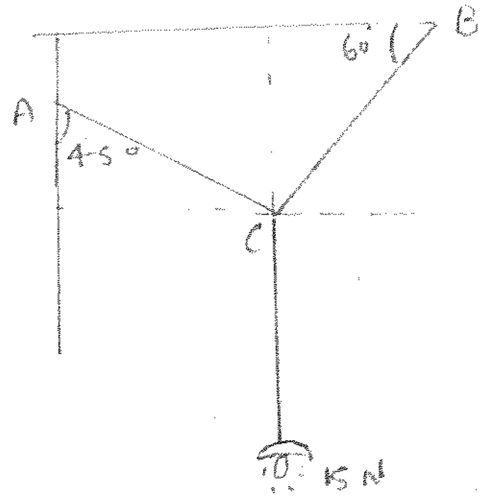
* The algebraic sum of the horizontal component of forces acting on the body is zero. i.e $\Sigma H = 0$.
* The algebraic sum of the vertical component of forces acting on the body is zero. i.e $\Sigma V = 0$.
* The algebraic sum of the moments of all the forces about any point is zero. i.e $\Sigma M = 0$.



Equilibrium of Coplanar concurrent force.

1] An electric light weighing 15N hangs from a point 'c' by two strings AC and BC. Determine the forces in the strings AC and BC.





By using Lami's theorem,

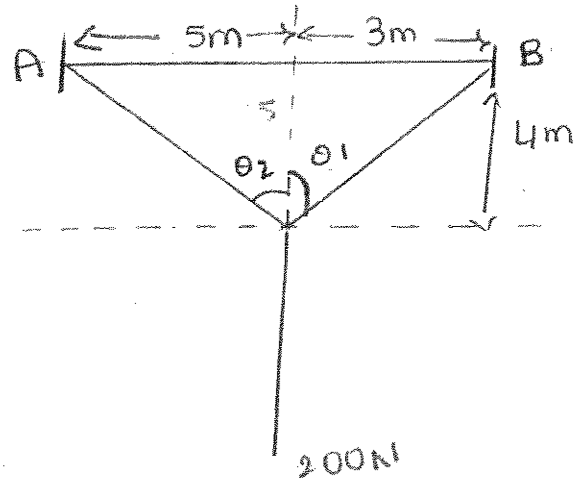
$$\frac{T_{AC}}{\sin 150} = \frac{15}{\sin 75} = \frac{T_{BC}}{\sin 135^\circ}$$

$$\therefore\ T_{AC} = \left(\frac{15}{\sin 75}\right) \times \sin 150 = 7.76\ N$$

(or) $T_{BC} = \frac{15}{\sin 75} \times \sin 135$

$$T_{BC} = \left(\frac{7.76}{\sin 150}\right) \sin 135^\circ = 10.98\ N$$

2] Two cables are tighted together At 'c'. and loaded as shown in the figure. Determine tensions in AC and BC.

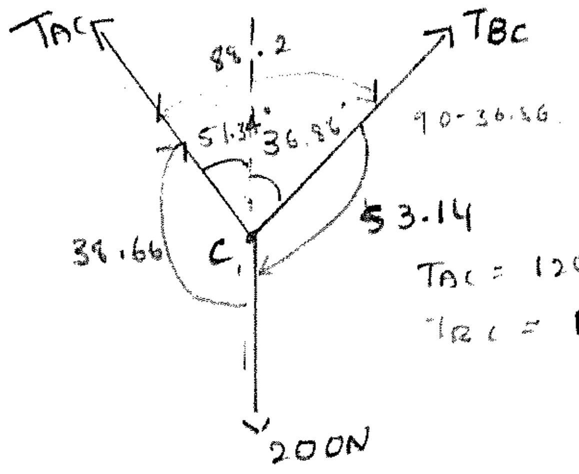


$\tan\theta_1 = \frac{3}{4}$ $\quad \theta_1 = \tan^{-1}(3/4)$

$\theta_1 = 36.86$

$\tan\theta_2 = \frac{5}{4}$ $\quad \theta_2 = \tan^{-1}(5/4)$

$\theta_2 = 51.34^\circ$



TAC = 120.06N = 160

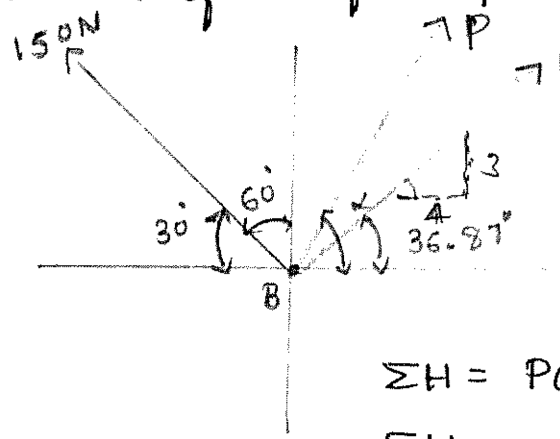
TBC = 156.25N

$$\frac{T_{AC}}{\sin 53.14} = \frac{200}{\sin 88.2} = \frac{T_{BC}}{\sin 38.66}$$

$$T_{AC} = \frac{200}{\sin 88.2} \times \sin 53.14 = 160.09N$$

$$T_{BC} = \frac{200}{\sin 88.2} \times \sin 38.66 = 125.54N$$

A particle B is in equillibrium. Action of 4 forces as shown in the figure, determine a magnitude and direction of the force p.



$\tan\theta = 3/4$

$\theta = 36.87°$

$$\Sigma H = P\cos\alpha + 100\cos 36.87 - 150\cos 30° = 0$$

$$\Sigma H = P\cos\alpha - 49.90.$$

$$P\cos\alpha = 49.90 \rightarrow (1)$$

$$\Sigma V = P\sin\alpha + 100\sin 36.87 + 150\sin 30° - 175$$

$$P\sin\alpha - 40$$

$$P\sin\alpha = 40 \rightarrow (2)$$

$\div$ 2 by 1

$$\frac{\cancel{P}\sin\alpha}{\cancel{P}\cos\alpha} = \frac{40}{49.90}$$

$$\tan\alpha = \frac{40}{49.90}$$

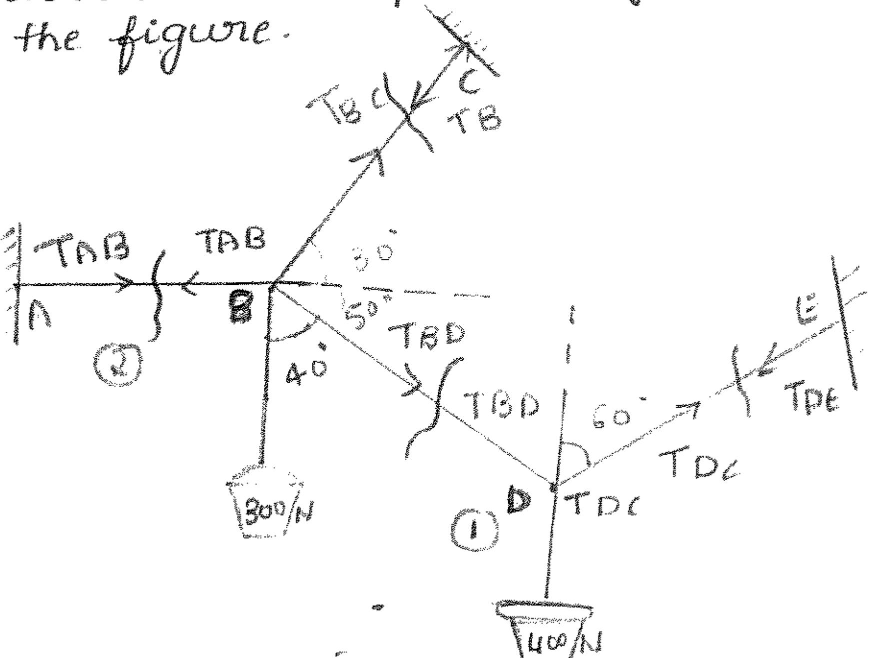
$$\alpha = \tan^{-1}\left(\frac{40}{49.90}\right)$$

$$\boxed{\alpha = 38.7^\circ}$$

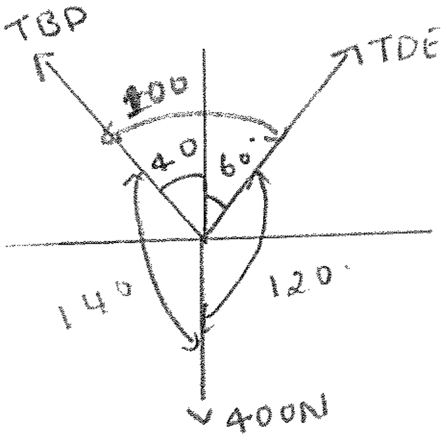
eq^n ① $P = \dfrac{49.90}{\cos 38.7}$

$$\boxed{P = 63.94\,N}$$

Two loads are suspended by rope system as shown in the figure.



①



Using Lami's Theorem

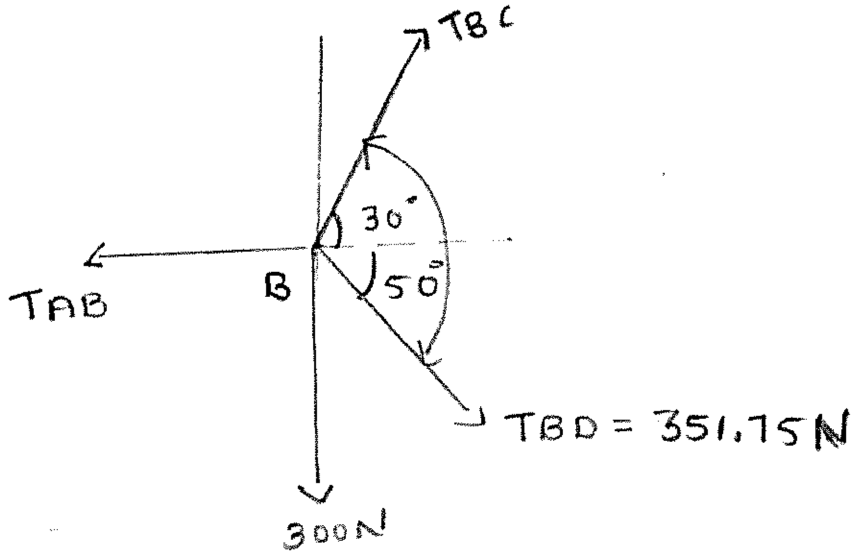
$$\frac{TBD}{\sin 120} = \frac{400}{\sin 100} = \frac{TDE}{\sin 140}$$

$$TBD = \frac{400}{\sin 100} \times \sin 120$$

$$\boxed{TBD = 351.75\,N}$$

$$T_{DE} = \frac{400}{\sin 100} \times \sin 140$$

$$\boxed{TDE = 261.08\,N}$$



$$\Sigma H = T_{BC}\cos 30^\circ + 351.75\cos 50^\circ - T_{AB} = 0 \rightarrow (1)$$

$$\Sigma V = T_{BC}\sin 30^\circ - 351.75\sin 50^\circ - 300 = 0 \rightarrow (2)$$

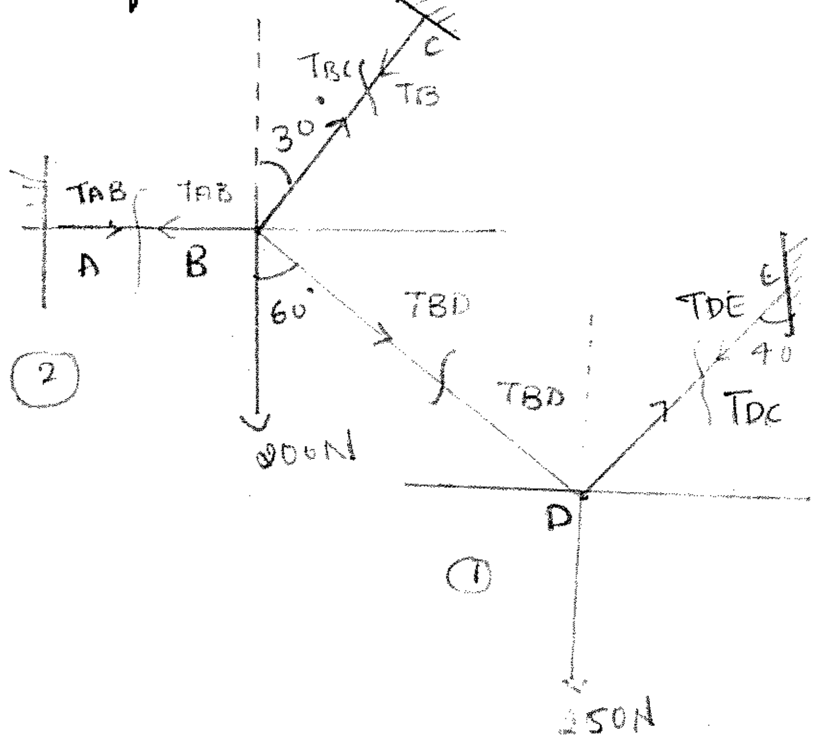
$$\boxed{T_{BB} = 1138.91\,N}$$

from eq (1) $T_{AB} = (1138.91)\cos 30^\circ + 351.75\cos 50^\circ$

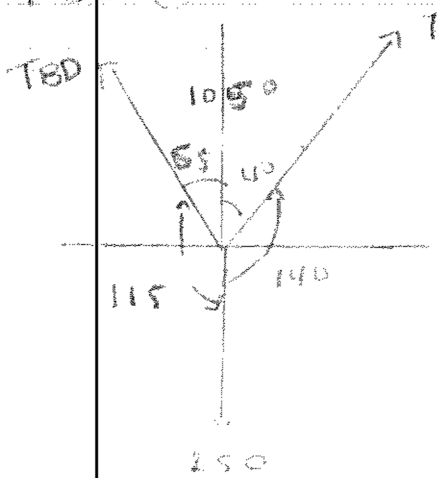
$$\boxed{T_{AB} = 1212.4\,N}$$

July 2023

A system of connected flexible cables shown in the figure. is supporting two vertical forces 200N and 250N at points B & D ~~resp~~ respectively. Determine the forces in the various segments of the cable.



FBD ①



$$\frac{TBD}{\sin 140} = \frac{250}{\sin 105} = \frac{TDF}{\sin 115}$$

$$\frac{TBD}{\sin 140} = \frac{250}{\sin 105}$$

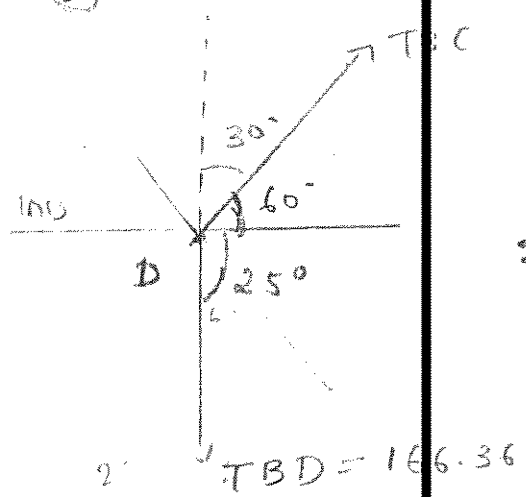
$$TBD = \frac{250}{\sin 105} \times \sin 140$$

$$\boxed{TBD = 166.36\,N}$$

$$TDE = \frac{250}{\sin 105} \times \sin 115$$

$$\boxed{TDE = 234.56\,N}$$

FBD = ②



$\Sigma H = TBC \cos 60° + 166.36 \cos 25 - TAB = 0 \rightarrow ①$

$\Sigma V = TBC \sin 60° - 166.36 \sin 25 - 200 = ②$

$$\boxed{TBC = 312.12}$$

$TAB = (312.12)\cos 60° + 166.36 \cos 25$

$$\boxed{TAB = 306.83\,N}$$

June ...

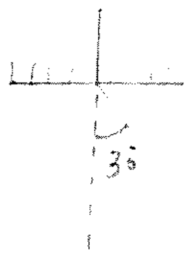
⑥

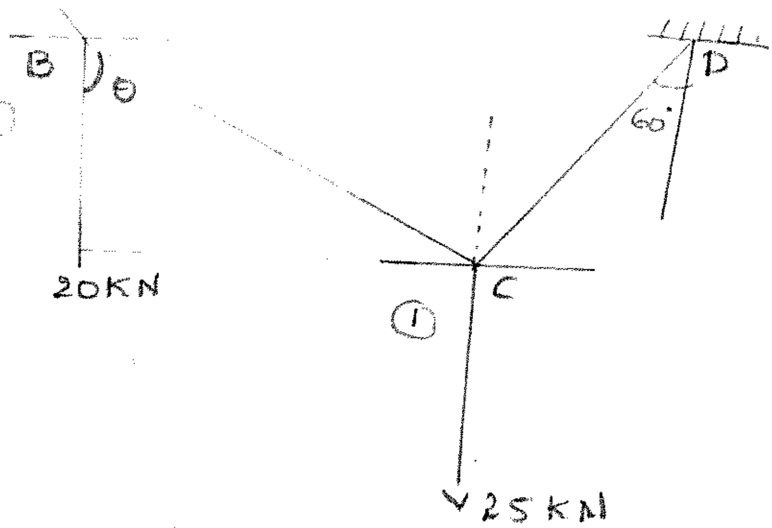
$T_{AB} = 38.97$ KN

$T_{BC} = 23.84$ KN

$T_{CD} = 22.5$ KN

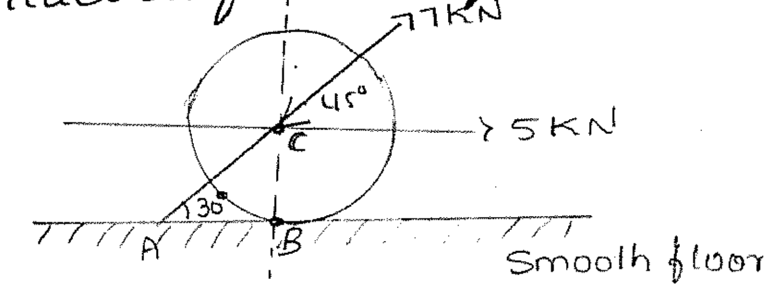
$\theta = 54.78°$

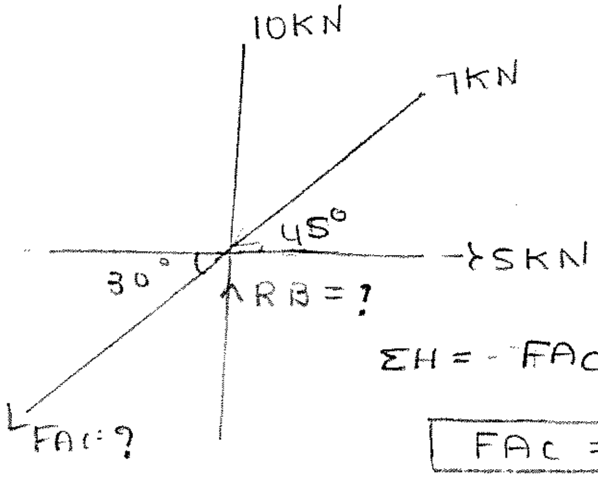




FBD①

A roller of weight 10KN rest on a smooth horizon floor and is connected to the floor by the bar AB as shown in the figure. Determine the force in the bar AC and reaction from the floor.





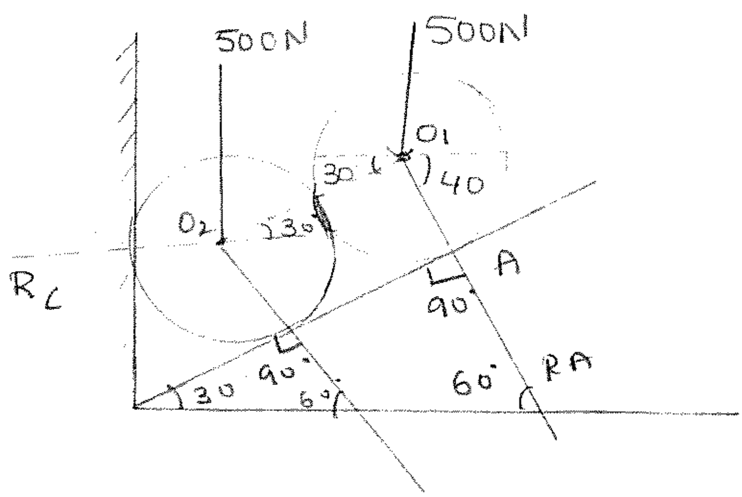
$\Sigma H = -FAC \cos 30 - 7\cos 45° - 5$

$\boxed{FAC = 0.058\ N}$

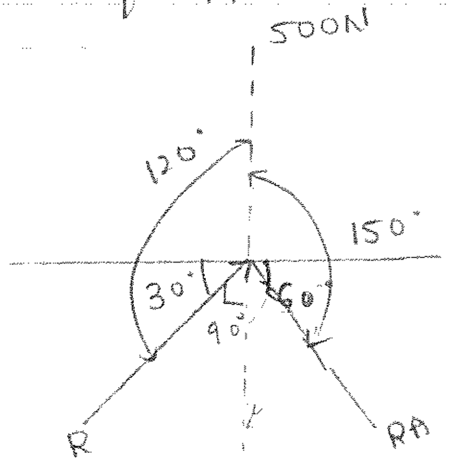
$\Sigma V = -10 - 7\sin 45° + RB - 0.058 \sin 30$

$\boxed{RB = 14.97N}$

The identical roller each of weight 500N are supported by a inclined plane and vertical wall as shown in the figure. Assuming smooth surface, find the ~~rediar~~ reaction induced at the point A B & C.



FBD of upper roller



Using Lami's theorem

$$\frac{R_A}{\sin 120} = \frac{500}{\sin 90} = \frac{R}{\sin 150}$$

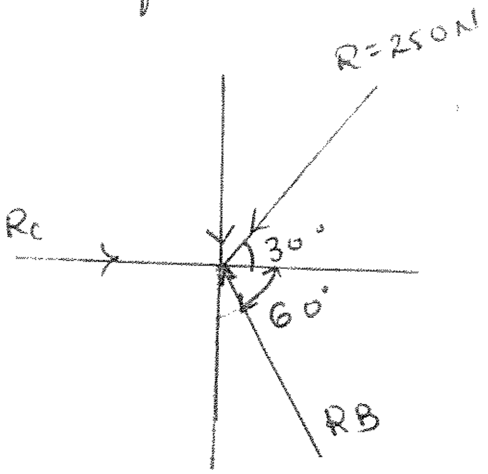
$$R_A = \frac{500}{\sin 90} \times \sin 120$$

$$\boxed{R_A = 433N}$$

$$R = \frac{500}{\sin 90} \times \sin 150$$

$$\boxed{R = 250N}$$

FBD of lower roller



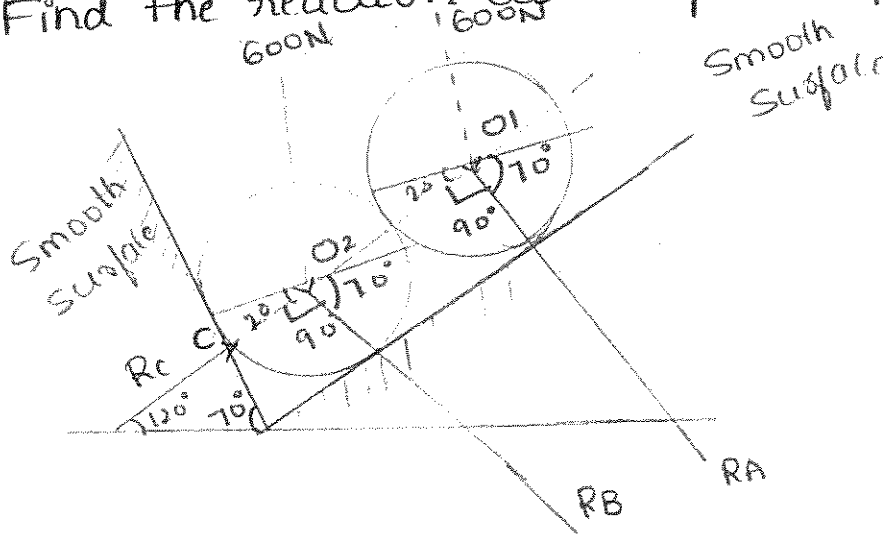
$$\Sigma H = -250\cos 30° + R_C - R_B \cos 60° = 0 \quad \text{—} (1)$$

$$\Sigma V = -250\sin 30° + R_B \sin 60° - 500$$

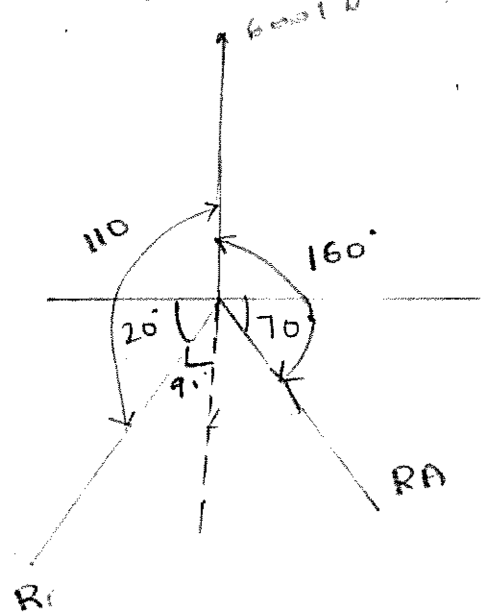
$$\boxed{R_B = 721.68}$$

$$\boxed{R_C = 577.34}$$

Two identical cylinder each weighing 600N are resulting over inclined planes as shown in the fig. Find the reaction at the points A, B & C.



FBD of upper roller



using lami's theorem

$$\frac{RA}{\sin 110} = \frac{600}{\sin 90} = \frac{R}{\sin 160}$$

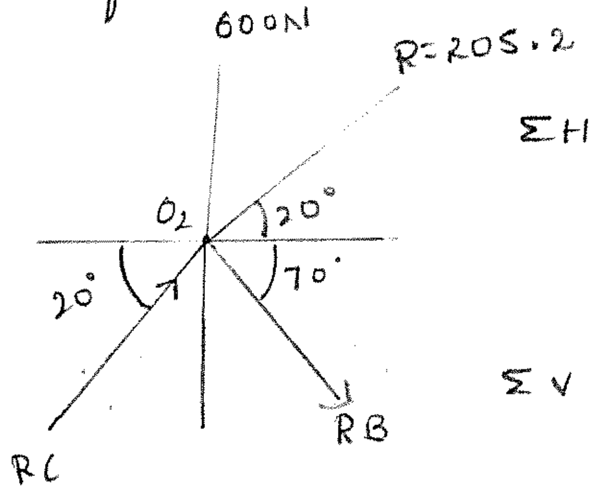
$$RA = \frac{600}{\sin 90} \times \sin 110$$

$$\boxed{RA = 563.82 \text{ N}}$$

$$R = \frac{600}{\sin 90} (\sin 160)$$

$$\boxed{R = 205.2 \text{N}}$$

FBD of lower roller



$$\Sigma H = -R_B \cos 70° + R_C \cos 20° - 205.21 \cos 20$$

$$-RB \cos 70° + R_C \cos 20° = 205.21 \cos 20 \rightarrow (1)$$

$$\Sigma V = RB \sin 70° + Rc \sin 20° - 205.21 \sin 20 - 600$$

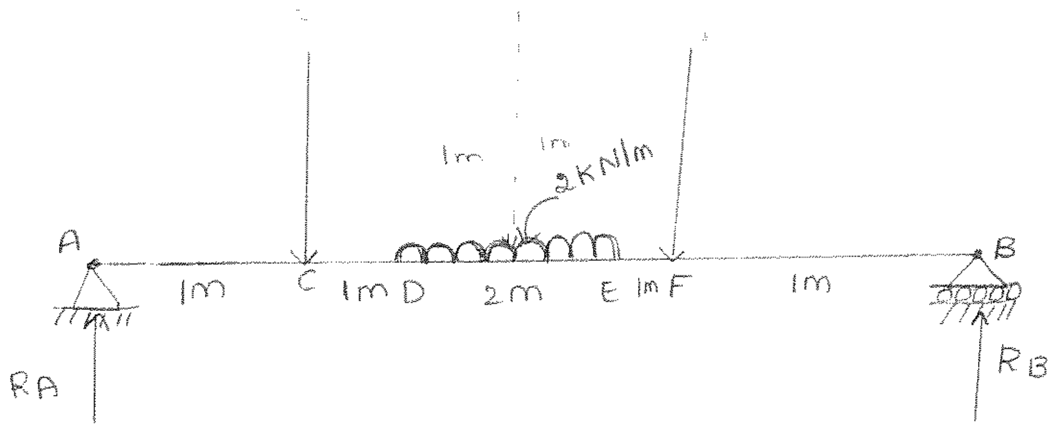
$$RB \sin 70 + Rc \sin 20° = 670.18 \rightarrow (2)$$

$$\boxed{R_B = 563.82 \text{N}}$$

$$\boxed{R_C = 410.42 \text{N}}$$

Equilibrium of Coplanar parallel force System

1] Determine support reaction at the support further following beam. show in the figure.



$\Sigma V = 0$

$R_A - 3 - (2\times2) - 6 + R_B = 0$

$R_A + R_B = 3 + 4 + 6$

$R_A + R_B = 10 + 3$

$\boxed{R_A + R_B = 13}$

$\Sigma M_A = 0$

$-(R_B \times 6) + (3\times1) + (2\times2) \cancel{+} \times 3 \cancel{+} (6\times5) = 0$

$(R_B \times 6) = 3 + 4 \times 3 + 30$

$R_B \times 6 = 45$

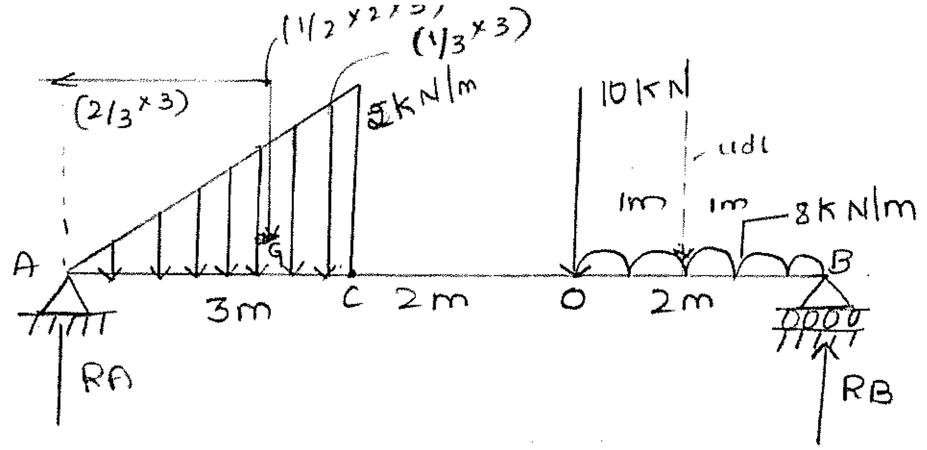
$R_B = \frac{45}{6}$ $\boxed{R_B = 7.5}$ KN

$R_A + R_B = 13$

$R_A = 13 - 7.5$

$\boxed{R_A = 5.5}$ KN

②



$\Sigma V = 0$

$RA + RB = \left(\frac{1}{\cancel{2}} \times \cancel{2} \times 3\right) + 10 + (8 \times 2)$

$\boxed{RA + RB = 29}$

$\Sigma M_A = 0$

$(RB \times 7) = \left(\frac{1}{\cancel{2}} \times \cancel{2} \times 3\right) \times \left(\frac{2}{\cancel{3}} \times \cancel{3}\right) + (10 \times 5) + (8 \times 2 \times$

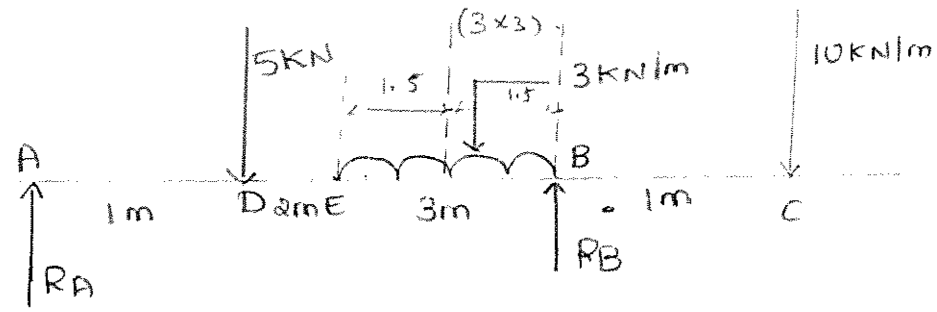
$RB \times 7 = 152$

$\boxed{RB = 21.7}$ KN

$RA = 29 - 21.7$

$\boxed{RA = 7.3 \text{ KN}}$

③



$\Sigma V = 0$

$(RA + RB) = 5 + (3 \times 3) + 10$

$\boxed{RA + RB = 24}$

$\Sigma M_A = 0$

$$(R_B \times 6) = (5\times1) + (3\times3)\times 4.5 + (10\times7)$$

$$R_B \times 6 = 5 + \cancel{36} + 70 + 40.5$$

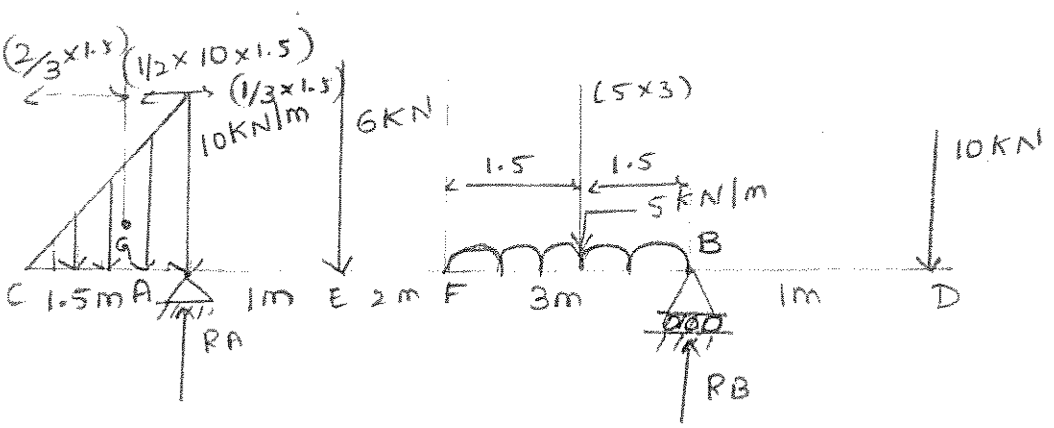
$$R_B = 115.5$$

$$\boxed{R_B = 19.25\ KN}$$

$$R_A = 24 - 19.25$$

$$\boxed{R_A = 4.75\ KN}$$

(4)



$\Sigma V = 0$

$$R_A + R_B = (\tfrac{1}{2}\times10\times1.5) + 6 + (5\times3) + 10$$

$$\boxed{R_A + R_B = 38.5}$$

$\Sigma M_A = 0$

$$(R_B \times 6) + \left[\left(\frac{1}{2}\times \overset{5}{\cancel{10}} \times 1.5\right)\times\left(\frac{1}{3}\times \overset{0.5}{\cancel{1.5}}\right)\right] \pm (6\times1) + (5\times3)\times 4.5 + (10\times7)$$

$$R_B \times 6 + 3.75 = 143.5$$

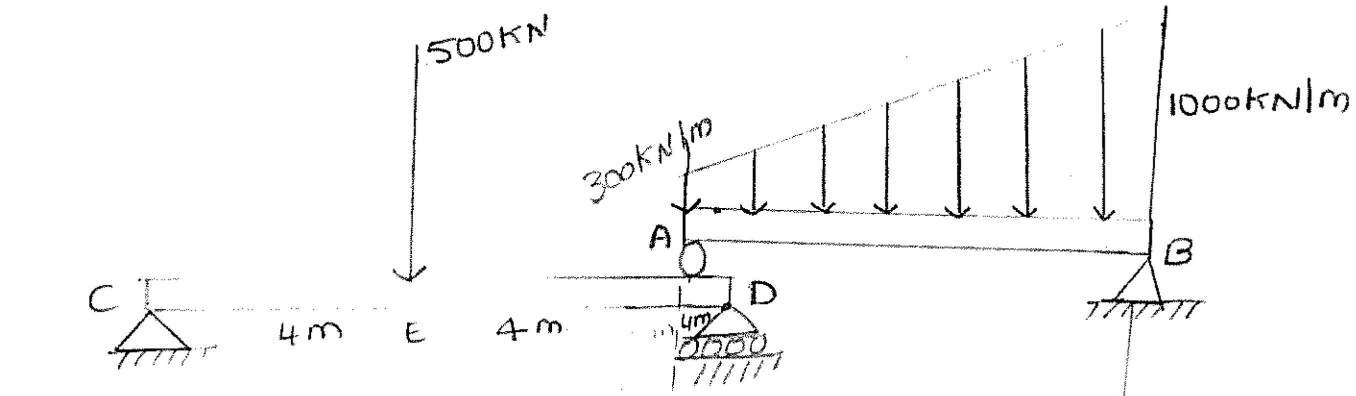
$$R_B = \frac{143.5 - 3.75}{6}$$

$$\boxed{R_B = 23.29\ KN}$$

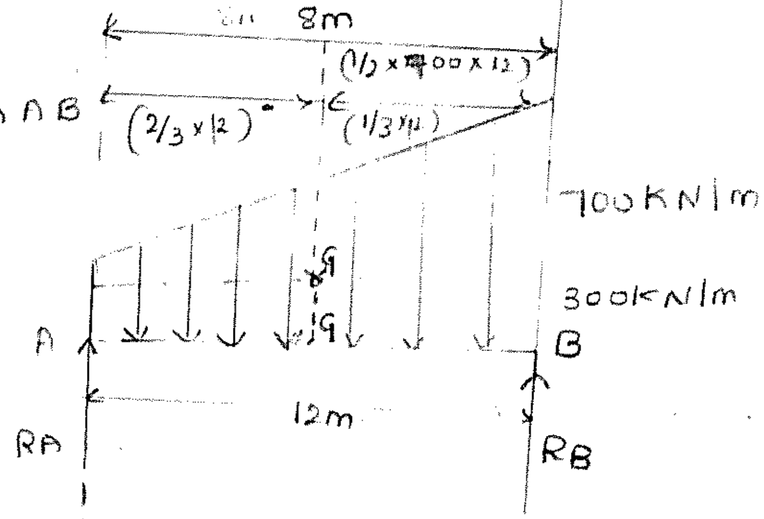
$$R_A = 38.5 - 23.29$$

$$\boxed{R_A = 15.21\ KN}$$

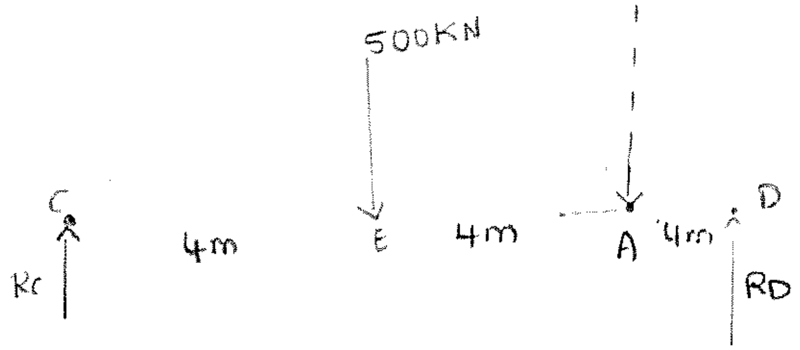
(5) Determine the reactions at ...



Beam AB



Beam CD



$\Sigma V = 0$

$R_A + R_B = 12 \times 300 + (1/2 \times 700 \times \overset{6}{12})$

$R_A + R_B = 3600 + 4200$

$\boxed{R_A + R_B = 7800}$

$\Sigma M_A = 0$

$(R_B \times 12) = (300 \times 12) \times 6 + (700 \times 6) \times 8 = 0$

$R_B \times 12 = 21,600 + 33600 = 0$

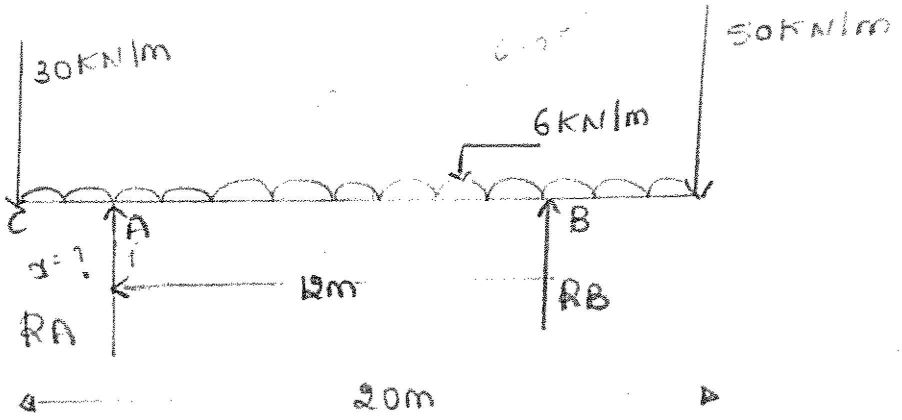
$\boxed{R_B = 4,600 \text{ KN}}$

$R_A + R_B = 7800$

$R_A = 7800 - 4600$

$\boxed{R_A = 3200\ kN}$

6] A beam shown in the figure. Determine the location of the two support. Show that both the reaction are equal. [i.e $RA = RB$].



$\Sigma V = 0$

$R_A + R_B = 30 + (6 \times 20) + 50$

$\boxed{R_A + R_B = 200}$

But $R_A = R_B$ is given

$2R_B = 200$ $\quad R_B = 100\,KN$

$R_A = 100\,KN$

$\Sigma M_A = 0$

$$(\overset{100}{R_B} \times 12) + (30 \times x) = (6 \times 20)(10 - x) + (50 \times 20 - x)$$

$$(100 \times 12) + 30x = 120(10 - x) + 50(20 - x)$$

$$\cancel{1200} + 30x = \cancel{1200} - 120x + 1000 - 50x$$

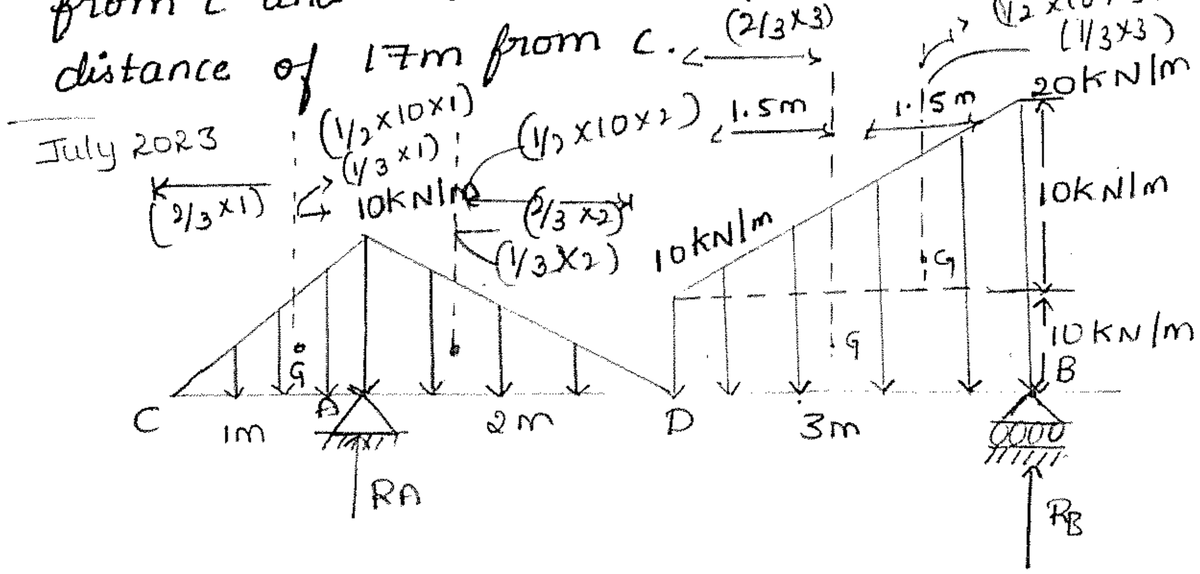
$$30x = -170x + 1000$$

$$200x = 1000 \qquad x = \frac{\overset{5}{\cancel{1000}}}{\cancel{200}}$$

$$\boxed{x = 5m}$$

The first support is located at a distance of 5m from c and the second support is located at a distance of 17m from c.

July 2023



$\Sigma V = 0$

$$R_A + R_B = \left(\frac{1}{2} \times 10 \times 1\right) + \left(\frac{1}{2} \times 10 \times 2\right) + (10 \times 3) + (1/2 \times 10 \times 3)$$

$$\boxed{R_A + R_B = 60}$$

$\Sigma M_A = 0$

$$(R_B \times 5) + \left(\frac{1}{\cancel{2}} \times \overset{5}{\cancel{10}} \times 1\right) + (1/3 \times 1) = \left(\frac{1}{\cancel{2}} \times 10 \times \cancel{2}\right) \times (1/3 \times 2)$$

$$+ (10 \times 3) \times 3.5 + \left(\frac{1}{\cancel{2}} \times \overset{5}{\cancel{10}} \times 3\right) \times 2 + \frac{2}{\cancel{3}} \times \cancel{3}$$

$$R_B 5 + 5.33 = 143.66$$

$$R_B 5 = 143.66 - 5.33$$

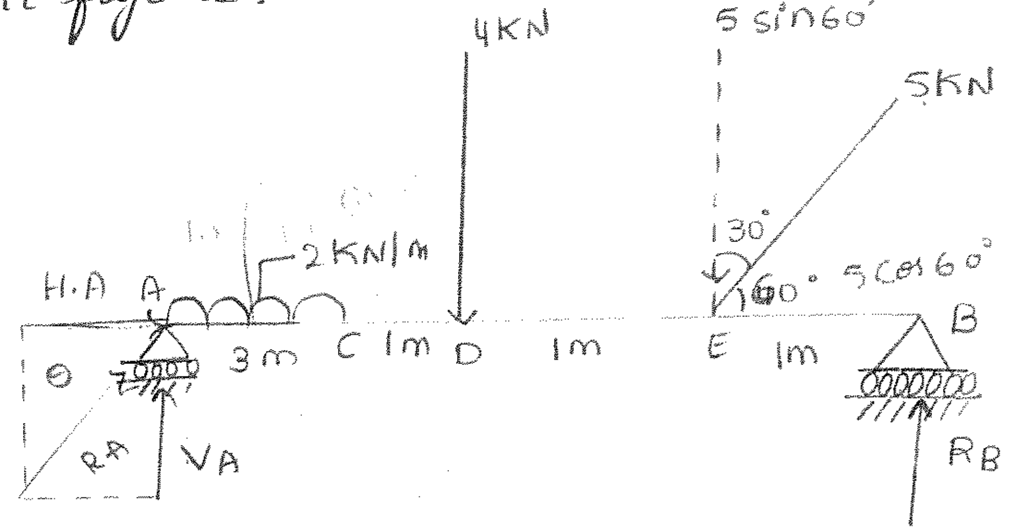
$$\boxed{R_B = 34\ kN}$$

$$R_A = 60 - 34$$

$$\boxed{R_A = 26\ kN}$$

Equilibrium of Coplanar Non-concurrent of force system :-

Find the support reaction at the enged support and roller support for the following beams shown in the figure.



$\Sigma H = 0$

$HA - 5\cos 60 = 0$

$HA = 5\cos 60$

$HA = 2.5\ KN$

$\Sigma V = 0$

$V_A + R_B = (2\times3)+4+5\sin 60$

$V_A + R_B = 14.33$

$\Sigma M_A = 0$

$(R_B \times 6) = (2\times3)\times1.5 + (4\times4) + (5\sin 60 \times 5)$

$\boxed{R_B = 7.775\ KN}$

$VA + RB \doteq 14.33$

$VA = 14.33 - 7.78$

$\boxed{V_A = 6.55\ KN}$

$$R_A = \sqrt{(H_A)^2 + (V_A)^2}$$

$$R_A = \sqrt{(2.5)^2 + (6.55)^2}$$

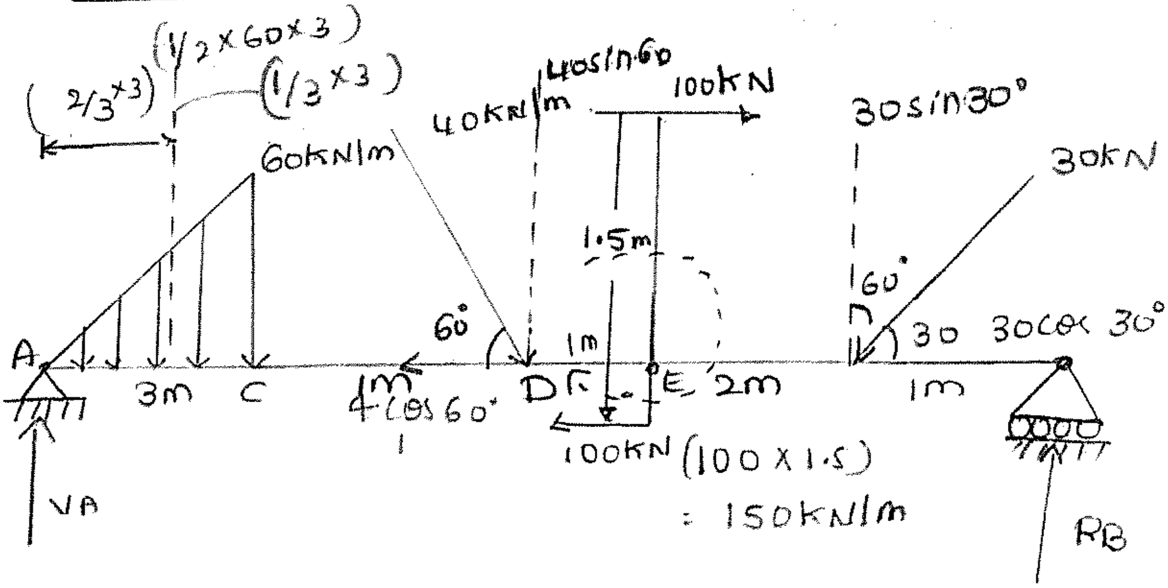
$$\boxed{R_A = 7.01\ KN}$$

$$\tan\theta = \frac{V_A}{H_A}$$

$$\theta = \tan^{-1}\left(\frac{6.55}{2.5}\right)$$

$$\boxed{\theta = 69.10^\circ}$$

(2)



$$\Sigma H = 0$$

$$H_A + 4\cos 60 - 30\cos 30 = 0$$

$$H_A = 5.98\ KN$$

$$\Sigma V = 0$$

$$V_A + R_B = \frac{1}{\cancel{2}} \times 60 \times 3 + 40\sin 60 + 30\sin 30$$

$$V_A + R_B = 139.64$$

$$\Sigma M_A = 0$$

$$(R_B \times 8) = \left(\frac{1}{2} \times \overset{30}{60} \times 3\right) \times \left(\frac{2}{3}\times 3\right) + (40\sin 60 \times 4) +$$

$$150 + 30\sin 30 \times 7$$

$$\boxed{R_B = 71.69\ KN}$$

$$V_A + R_B = 139.64$$

$$V_A = 139.64 - 71.69$$

$$\boxed{V_A = 67.95 \text{ KN}}$$

$$R_A = \sqrt{(H_A)^2 + (V_A)^2}$$

$$R_A = \sqrt{(5.98)^2 + (67.95)^2}$$

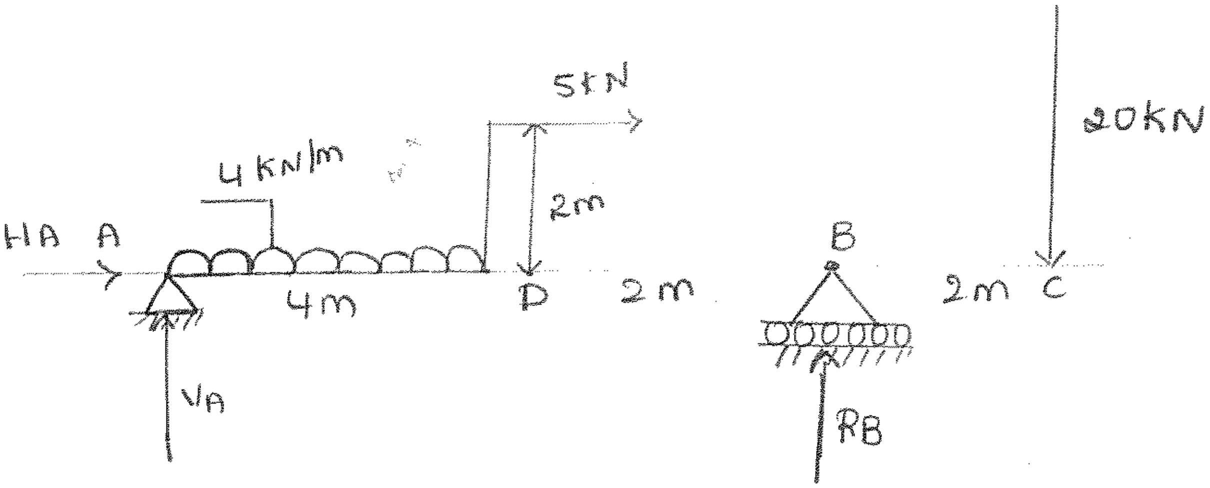
$$\boxed{R_A = 68.21 \text{ KN}}$$

$$\tan\theta = \frac{V_A}{H_A}$$

$$\theta = \tan^{-1}\left(\frac{67.95}{5.98}\right) \qquad \boxed{\theta = 84.97^\circ}$$

(4)

(3)



$$\Sigma H = 0$$

$$H_A + 5 = 0$$

$$\boxed{H_A = -5 \text{ KN}}$$

$$\Sigma V = 0$$

$$V_A + R_B = (4 \times 4) + 20$$

$$V_A + R_B = \underline{\underline{36}}$$

$$\Sigma M_A = 0$$

$$(R_B \times 6) = (4 \times 4) \times 2 + (5 \times 2) + (20 \times 8)$$

$$R_B = 33.67 \ \underline{\underline{\text{KN}}}$$

$V_A + R_B = 36$

$V_A = 36 - 33.67$

$\boxed{V_A = 2.33\ KN}$

$R_A = \sqrt{(H_A)^2 + (V_A)^2}$

$R_A = \sqrt{(5)^2 + (2.33)^2}$

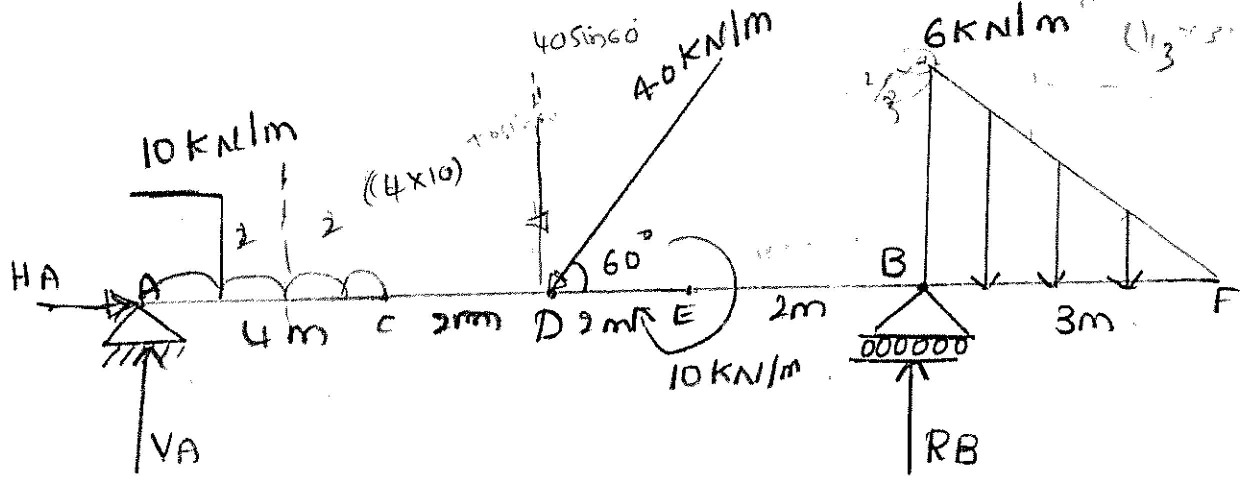
$\boxed{R_A = 5.51\ KN}$

$\tan\theta = \frac{V_A}{H_A}$

$\theta = \tan^{-1}\left(\frac{2.33}{5}\right)$

$\boxed{\theta = 24.98^\circ}$

(4)



$\Sigma H = 0$

$H_A - 4\cos 60^\circ = 0$

$H_A = \underline{20\ KN}$

$\Sigma V = 0$

$V_A + R_B = (10 \times 4) + 40\sin 60^\circ + \left(\frac{1}{2} \times 6 \times 3\right)$

$V_A + R_B = \underline{83.64}\ KN$

$\Sigma M_A = 0$

$(R_B \times 10) = (10 \times 4) \times 2 + (40\sin 60 \times 6) + (\frac{1}{2} \times \cancel{6} \times 3) \times (10 + \frac{1}{\cancel{3}} \times \cancel{3}) + 10$

$\boxed{R_B = 39.68\ KN}$

$V_A + R_B = 83.64$

$V_A = 83.64 - 39.68$

$V_A = 43.96\ KN$

$R_A = \sqrt{(H_A)^2 + (V_A)^2}$

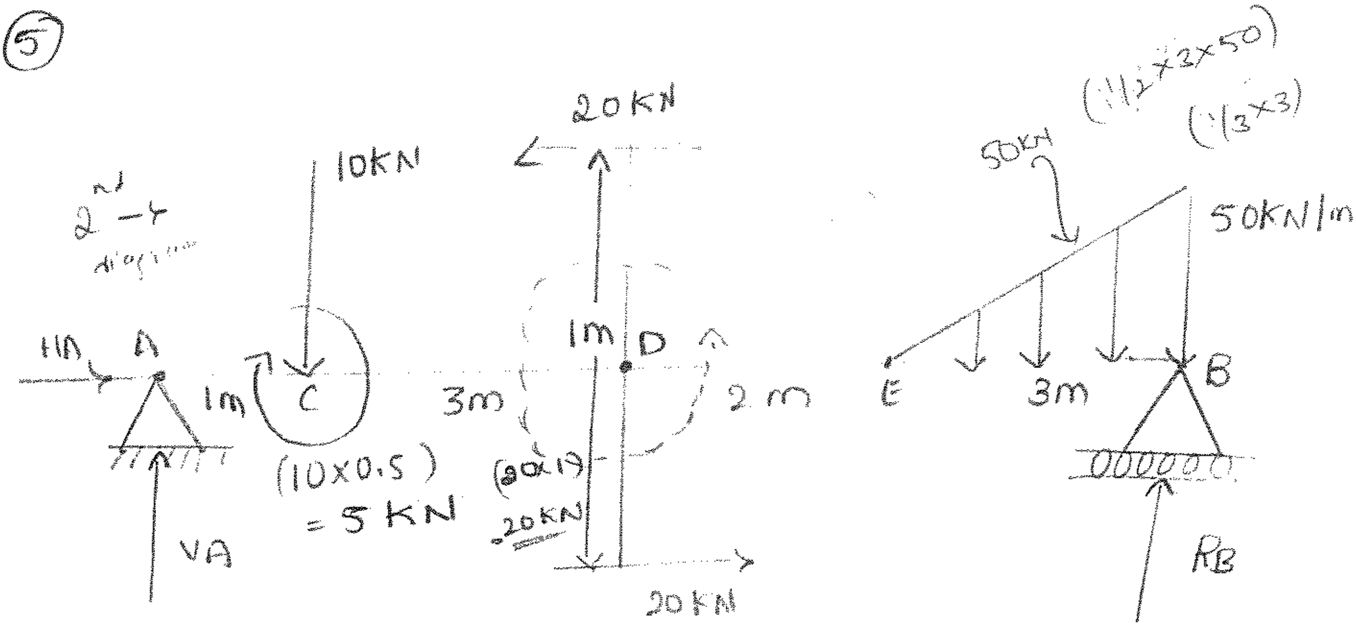
$R_A = \sqrt{(20)^2 + (43.96)^2}$

$R_A = 48.29\ KN$

$\theta = \tan^{-1}\left(\frac{43.96}{20}\right)$

$\boxed{\theta = 65.53^\circ}$

(5)



$\Sigma H = 0$

$\boxed{H_A = 0}$

$\Sigma V = 0$

$V_A + R_B = 10 + (1/2 \times 50 \times 3)$

$V_A + R_B = 85\ KN$

$\Sigma M_A = 0$

$(R_B \times 9) + 20 = (10 \times 1) + 5 + (1/2 \times 50 \times 3) \times (6 + \frac{2}{3} \times 3)$

$\boxed{R_B = 66.11\ KN}$

$V_A + R_B = 85$

$V_A = 85 - 66.11$

$V_A = 18.89\ KN$

$R_A = \sqrt{(H_A)^2 + (V_A)^2}$

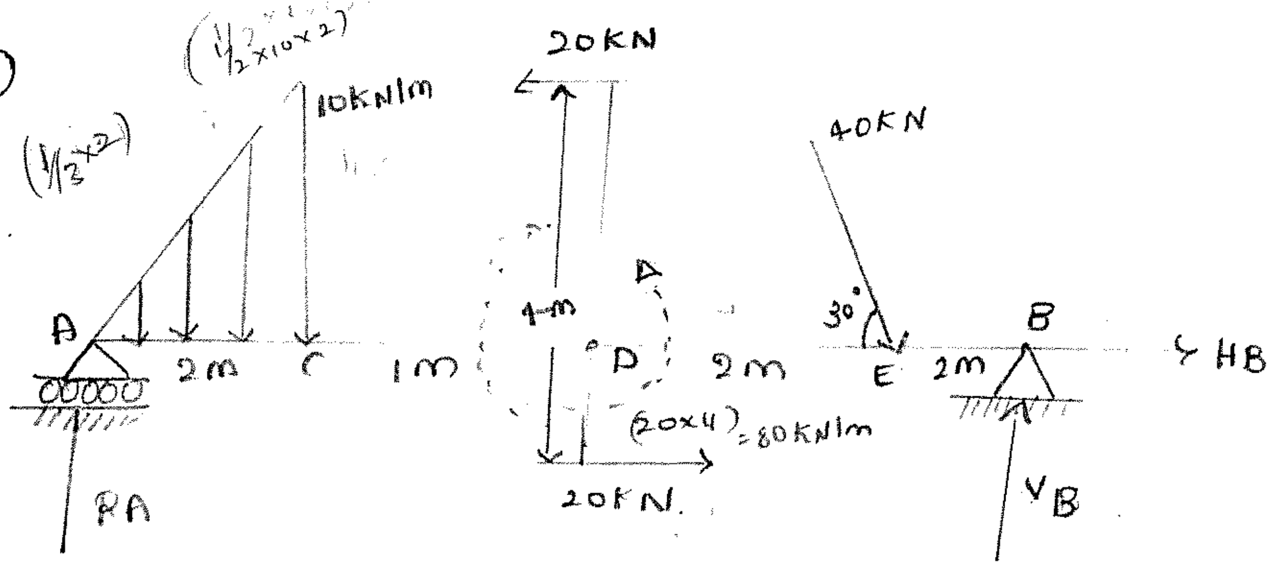
$R_A = \sqrt{0 + (18.89)^2}$

$R_A = 18.89\ KN$

$\theta = \tan^{-1}\left(\frac{18.89}{0}\right)$

$\boxed{\theta = \infty}$ Not defined

(6)



$\Sigma H = 0$

$HB + 40\cos 30$

$HB = -40\cos 30°$

$\boxed{HB = -34.64}$

$\Sigma V = 0$

$RA + V_B = \left(\frac{1}{2} \times 10 \times 2\right) + 40\sin 30$

$\boxed{RA + V_B = 30}$

$\Sigma M_B = 0$

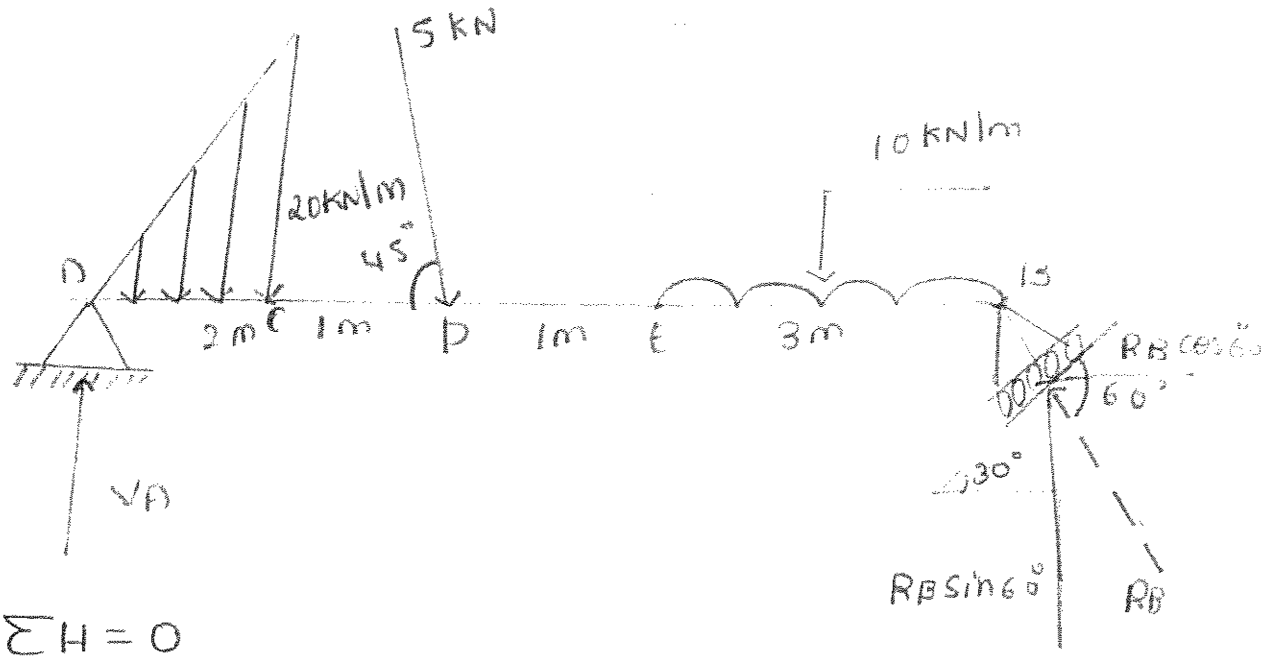
$(RA \times 7) = \left(\frac{1}{2} \times 10 \times 2\right) \times \left(5 + \frac{1}{3} \times 2\right) + 80 + (40\sin 30 \times 2)$

$RA \times 7 = 170.66$

$\boxed{RA = 25.24\ KN}$

x3)

(7)



$\Sigma H = 0$

$H_A + 5\cos 45° - R_B \cos 60° \rightarrow (1)$

$\Sigma V = 0$

$V_A + R_B \sin 60° = \left(\frac{1}{2} \times 20 \times 2\right) + (10 \times 3) + 5\sin 45$ 8]

$V_A + R_B \sin 60° = 53.54 \rightarrow (2)$

$\Sigma M_A = 0$

$(R_B \sin 60 \times 7) = \left(\frac{1}{2} \times 20 \times 2\right) \times \left(\frac{2}{3} \times 2\right) + (5\sin 45° \times 3)$
$+ (10 \times 3) \times 5.5$

$H_A = 13.15$
$V_A = 24.64$

$R_B \sin 60 (7) = 202.2$

$R_B \sin 60 = 28.89$

$R_B = \frac{28.89}{\sin 60}$

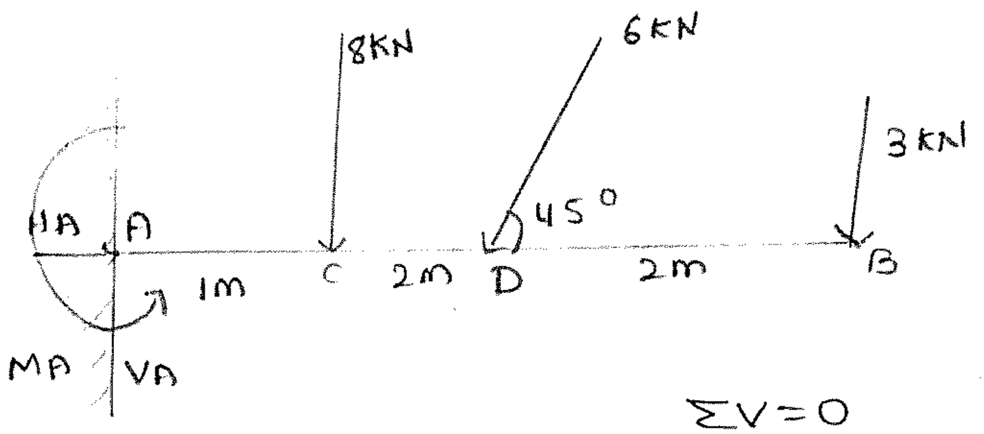
$\boxed{R_B = 33.35 \text{ kN}}$

$H_A + 5\cos 45 - R_B \cos 60°$

$H_A + 5\cos 45 - (33.35)\cos 60°$

$H_A + 5\cos 45 - 16.675$

8]



$\Sigma H = 0$

$HA - 6\cos 45 = 0$

$HA = 6\cos 45$

$HA = \underline{4.24\ KN}$

$\Sigma V = 0$

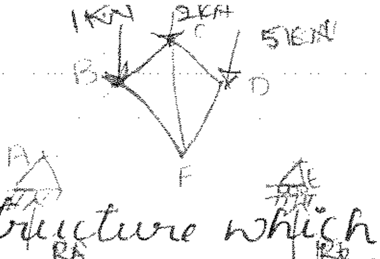
$VA = 8 + 6\sin 45 + 3$

$VA = \underline{15.24\ KN}$

$\Sigma MA = 0$

$MA = (8 \times 1) + (6\sin 45 \times 3) + (3 \times 5)$

$\boxed{MA = 35.72\ KN\text{-}m}$



TRUSSES

A truss [or a frame] is a structure which consists of number of members connected at their ends by pin joints to support an external load system.

A frame in which all the members lie in a single plane, then the stress is called a space truss [or space fram] plane truss, plane frame]
Roof trusses / Bridge trusses

Ex :- Tripods and Transmission.

The members of the truss will be in either tension (T) or compression [C].

If all the members of a truss do not lie in a single plane, then the truss is called a space truss [space frame]

Ex :- Tripods and Transmission towers.

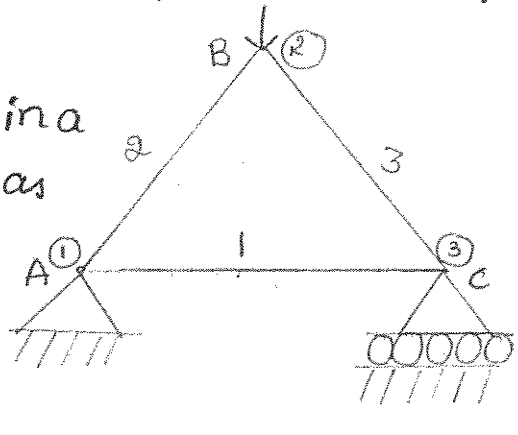
Classification of trusses :-

1] Perfect truss

2] Imperfect truss → Deficient truss; → Redundant truss.

1] Perfect truss :

A perfect frame is that it should retain its shape when load is applied at any joint in any direction.



The number of members in a perfect truss can be expressed as

$$n = (2j - 3)$$

n = number of members
J = number of joints

$\therefore \boxed{n = (2j-3)}$

$(n=3)\ (j=3)$

$(2j-3) = (2\times 3-3) = 3$

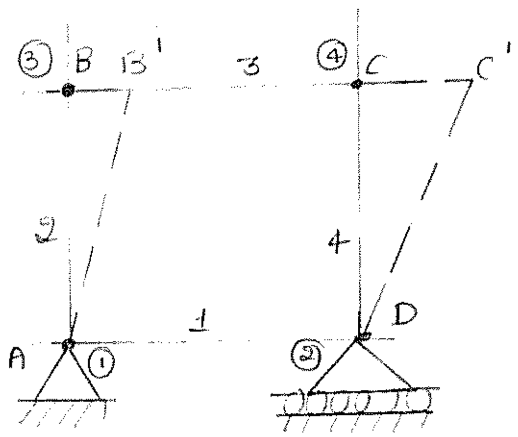
2] Imperfect truss :-

An imperfect truss is that which does not satisfy the equation, $n = (2j-3)$ i.e $n \neq (2j-3)$

a] Deficient truss :-

The number of members are less than $(2j-3)$ i.e $n < (2j-3)$

In this case the frame will not be stable, such that trusses cannot retain their shape when loaded.



$n = 4$

$j = 4$

$(2j-3) = (2\times 4-3)$

$= 5$

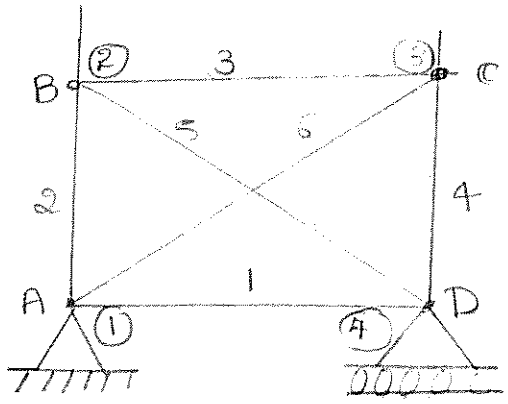
$\therefore \boxed{n < (2j-3)}$

b] Redundant truss :-

The number of members are more than $(2j-3)$ i.e $n > (2j-3)$

Redundant truss cannot be analysed by making use of the equations of equilibrium i.e $[\Sigma H = 0, \Sigma V = 0 \ \& \ \Sigma M = 0)$.

$\therefore$ A Redundant truss is statically indeterminate.



$n = 6$

$j = 4$

$(2j-3) = (2\times 4-3) = 5$

$\therefore \boxed{n > (2j-3)}$

Assumtions made in the analysis of trusses :-

1] The ends of the members are pin - Connected [hinged].

2] The truss is a perfect truss i.e. it should satisfy the equation (2j - 3) = n.

3] The loads act only at the joints.

④ Self weights of the members are neglected

5] Cross - section of the members are uniform.

To find forces in the members of the truss :-

Methods of Analysis :-

a] Method of joints

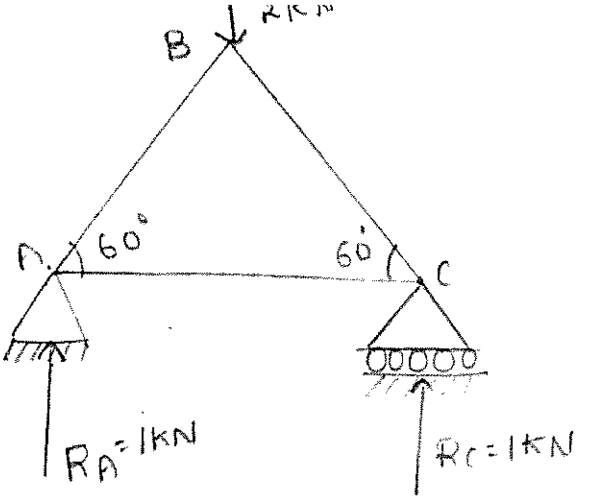
b] Method of sections.

a] Method of joints :-

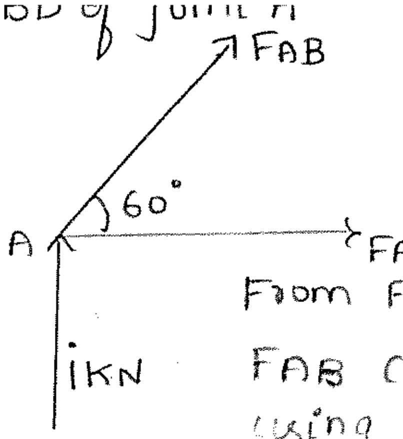
In this method each and every joint is selected and the unknown forces are then determined by the equation of equilibrium. The force meeting at a point and the loads acting if any constitue a system of concurrent forces.

∴ The equations of equilibrium $\Sigma H = 0$ and $\Sigma V = 0$ are used to determine the forces in the member.

While selecting the joint, care should be taken that at any instant the joint should not contain more than two members in which the forces are unknown.

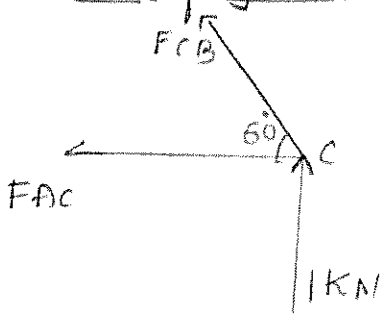


FBD of joint A



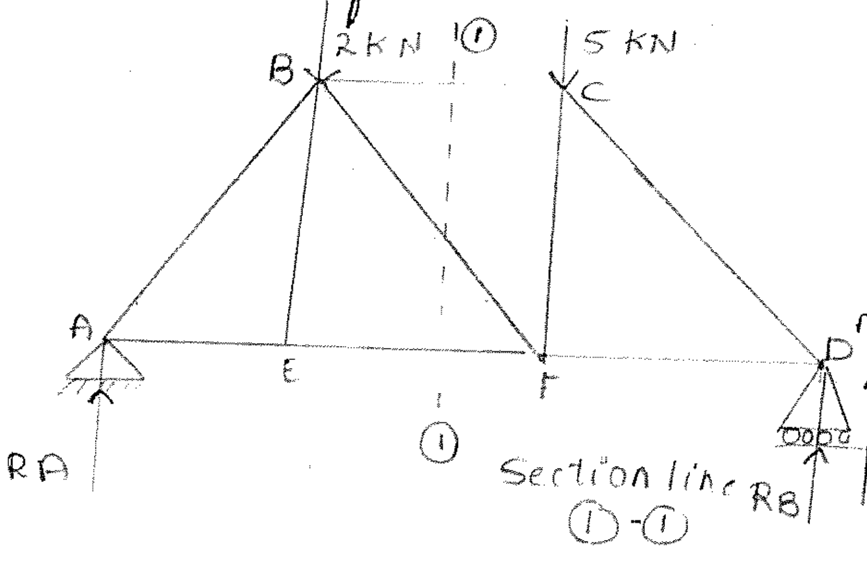
From FBD of joint A
F_{AB} can be found using $\Sigma V = 0$
F_{AC} can be found using $\Sigma H = 0$

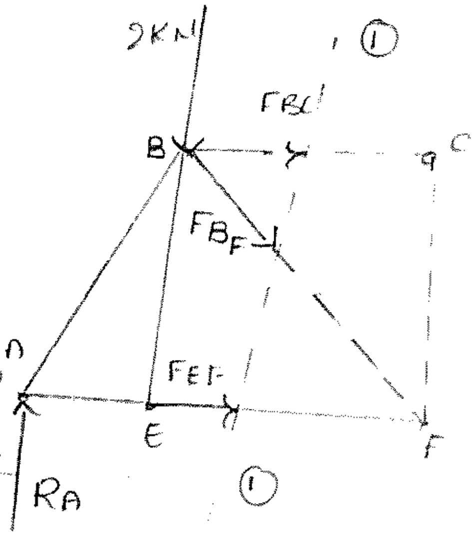
FBD of joint C



From FBD of joint C
F_{CB} can be found using $\Sigma V = 0$
F_{AC} can be found using $\Sigma H = 0$

b] Method of Sections :-





Consider FBD to the left of 1-1

To find force in member BC : $\Sigma M_F = 0 \quad \therefore F_{BC} =$

To find force in member EF : $\Sigma M_B = 0 \quad \therefore F_{EF} =$

To find force in member BF : use $\Sigma V = 0$ [or $\Sigma H = 0$)

$\therefore F_{BF} =$

In this method, a section line is passed through the members in which the forces are to be determined.

The Section line is drawn in which it should not cut more than three members in which the forces are unknown.

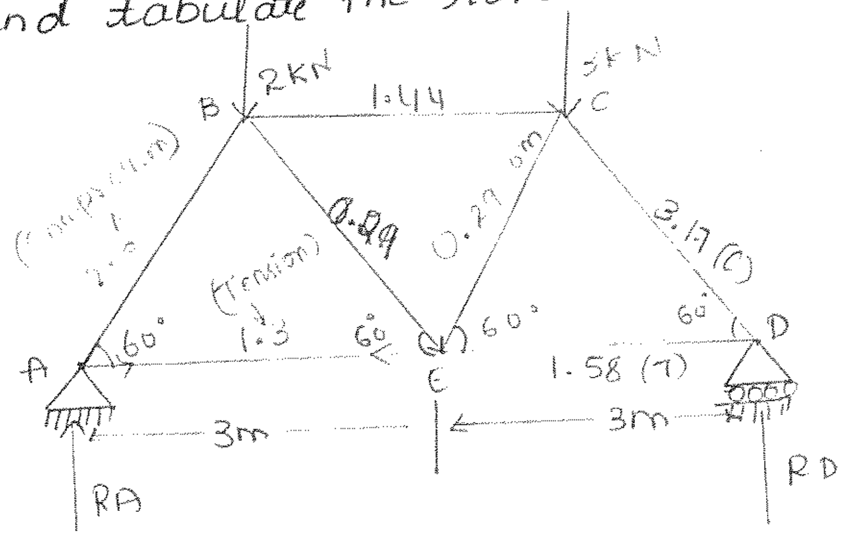
The unknown forces in the members are then determined by using the equations of equilibrium $\Sigma H = 0$, $\Sigma V = 0$ and $\Sigma M = 0$

(∵ The system of forces acting on either part of the truss constitutes a non-concurrent force system)

[The method of sections is used when force in only one member or the forces in very few members are to be determined.]

Analysis of trusses by Method of joints :-

01] Find the force in all the members of the truss or frame and tabulate the results.



To find the reaction, $\Sigma V = 0$

$R_A + R_B = 2 + 3$

$R_A + R_B = 5$

$\Sigma M_A = 0$

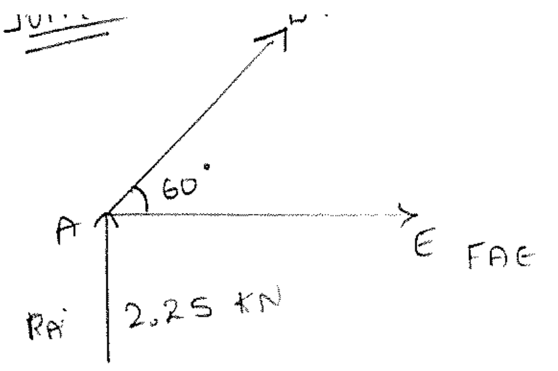
$(R_D \times 6) = (2 \times 1.5) + (3 \times 4.5)$

$\boxed{R_D = 2.75}$

$R_A = 5 - R_D$

$\boxed{R_A = 2.25}$

Joint A



$\Sigma V = 0$

$F_{AB} \sin 60 + 2.25 = 0$

$F_{AB} = \dfrac{-2.25}{\sin 60}$

$\boxed{F_{AB} = -2.6\,KN}$ (-) Compression

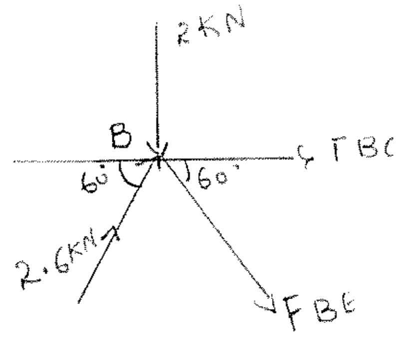
$\Sigma H = 0$

$F_{AB} + F_{AE}\ (\cos 60°) = 0$

$F_{AE} = \cos 60° (2.6)$

$\boxed{F_{AE} = 1.3\ KN}$ Tension

Joint B :



$\Sigma V = 0.$

$-2 + \sin 60 (2.6) - F_{BE} \sin 60 = 0$

$F_{BE} = \dfrac{-2 + 2.6 \sin 60}{\sin 60.}$

$\boxed{F_{BE} = 0.29\ KN}$ → (T)

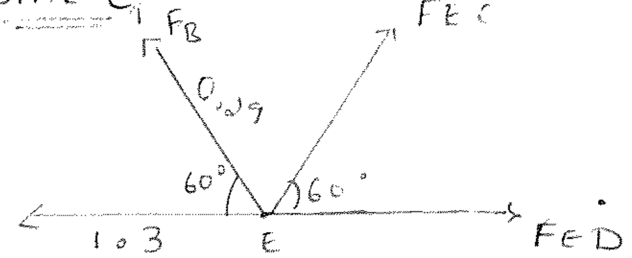
$\Sigma H = 0$

$F_{BC} + F_{BE} \cos 60 + 2.6 \cos 60° = 0$

$F_{BC} = -0.29 (\cos 60) - 2.6 \cos 60°$

$\boxed{F_{BC} = -1.44\ KN}$ → (C)

Joint E



$\Sigma V = 0.$

$0.29 \sin 60 + F_{EC} \sin 60 =$

$F_{EC} = \dfrac{-0.29 \sin 60}{\sin 60}$

$\boxed{F_{EC} = -0.29\ KN}$ → (C)

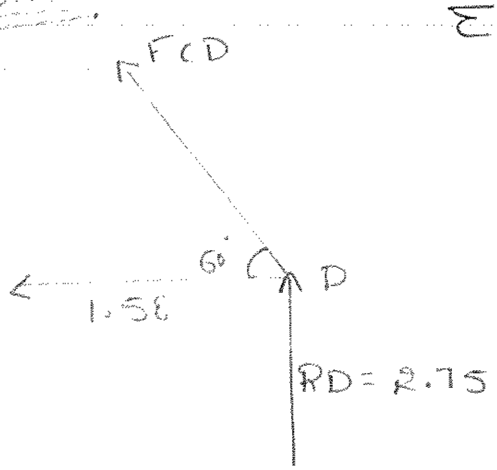
$\Sigma H = 0$

$F_{ED} + F_{EC} \cos 60 - 1.3 - 0.29 \cos 60 = 0$

$F_{ED} = +0.29 \cos 60° + 1.3 + 0.29 \cos 60°$

$\boxed{F_{ED} = 1.58\ KN}$ → (T)

Joint D



$\Sigma V = 0$

$F_{CD} \sin 60 + 2.75 = 0$

$F_{CD} = \frac{-2.75}{\sin 60}$

$\boxed{F_{CD} = -3.17\ KN} \rightarrow (C)$

$\Sigma H = 0$ [Check]

$-1.58 - F_{CD} \cos 60$

$-1.58 + 3.17 \cos 60$

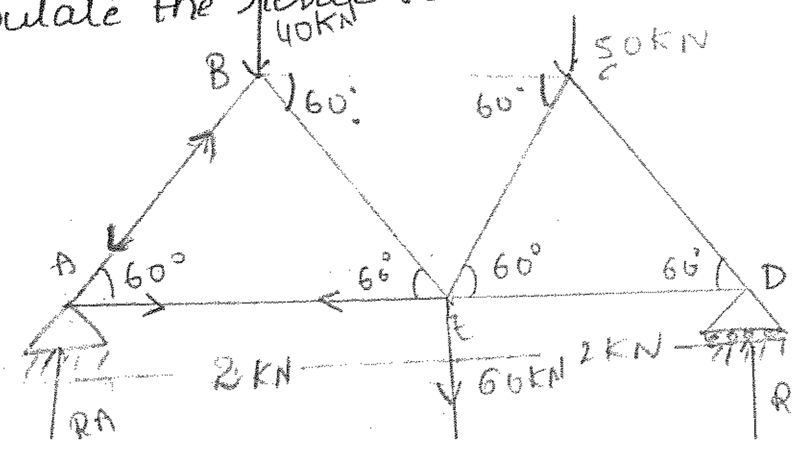
5×10^{-3}

$= 0.005$

$\therefore \Sigma H = 0$ Hence analysis is ~~zero~~. Correct.

Member	Force in KN	Nature
AB	2.6	Compression
BC	1.44	Compression
CD	3.17	Compression
AE	1.3	Tension
ED	1.58	Tension
BE	0.29	Tension
CE	0.29	Compression

Determine forces in all the members of the truss and tabulate the results.



To find reaction ;

$\Sigma V = 0$

$R_A + R_D = 40 + 50 + 60$

$\boxed{R_A + R_D = \cancel{42}\ 150}$

$\Sigma M_A = 0$

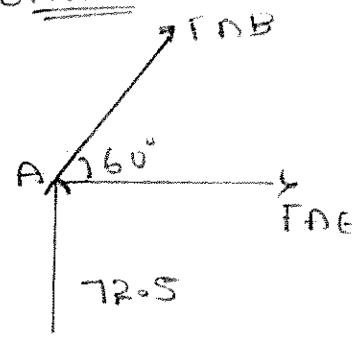
$(R_D \times 4) = (40 \times 1) + (50 \times 3) + (60 \times 2)$

$R_D = 77.5\ KN$

$R_A = 150 - 77.5$

$R_A = 72.5\ KN$

Joint A



$\Sigma V = 0$

$F_{AB} \sin 60° + 72.5$

$F_{AB} = \dfrac{-72.5}{\sin 60°}$

$\boxed{F_{AB} = -83.71}$ → (C)

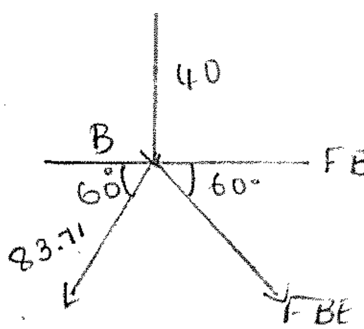
$\Sigma H = 0$

$F_{AE} + F_{AB} \cos 60° = 0$

$F_{AE} = 83.71 \cos 60°$

$\boxed{F_{AE} = 41.85\ KN}$ → (T)

Joint B :-



$\Sigma V = 0$

$-40 + 83.71 \sin 60 - F_{BE} \sin 60 = 0$

$-F_{BE} \sin 60 = 40 - 83.71 \sin 60$

$-F_{BE} = \dfrac{40 - 83.71 \sin 60}{\sin 60}$

$\boxed{F_{BE} = 37.5\ KN}$

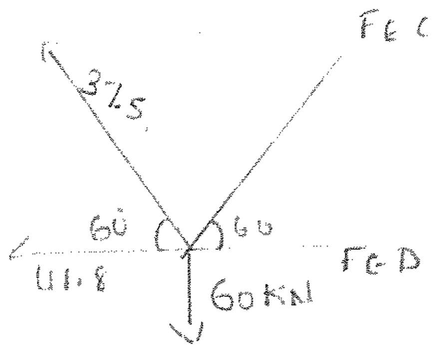
$\Sigma H = 0$

$F_{BC} + F_{BE} \cos 60 + 83.71 \cos 60^\circ = 0$

$F_{BC} = -37.5 \cos 60 - 83.71 \cos 60$

$\boxed{F_{BC} = -60.6 \text{ KN}}$

Joint E



$\Sigma V = 0$

$37.5 \sin 60^\circ + F_{EC} \sin 60 - 60$

$F_{EC} = \dfrac{60 - 37.5 \sin 60}{\sin 60}$ 3]

$F_{EC} = 31.76 \text{ (T)}$

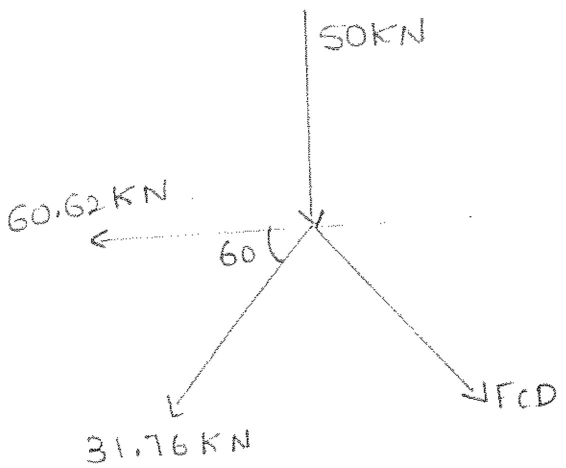
$\Sigma H = 0$

$F_{ED} - 41.85 + F_{EC} \cos 60 - 31.62 \cos 60 = 0$

$F_{ED} = 41.85 + 31.62 \cos 60 - 31.76 \cos 60$

$F_{ED} = 44.75 \text{ (T)}$

Joint C



$\Sigma V = 0$

$-50 - F_{CD} \sin 60 - 31.76 \sin 60 = 0$

$-F_{CD} = \dfrac{50 + 31.76 \sin 60}{\sin 60}$

$F_{CD} = -89.49 \text{ (C)}$

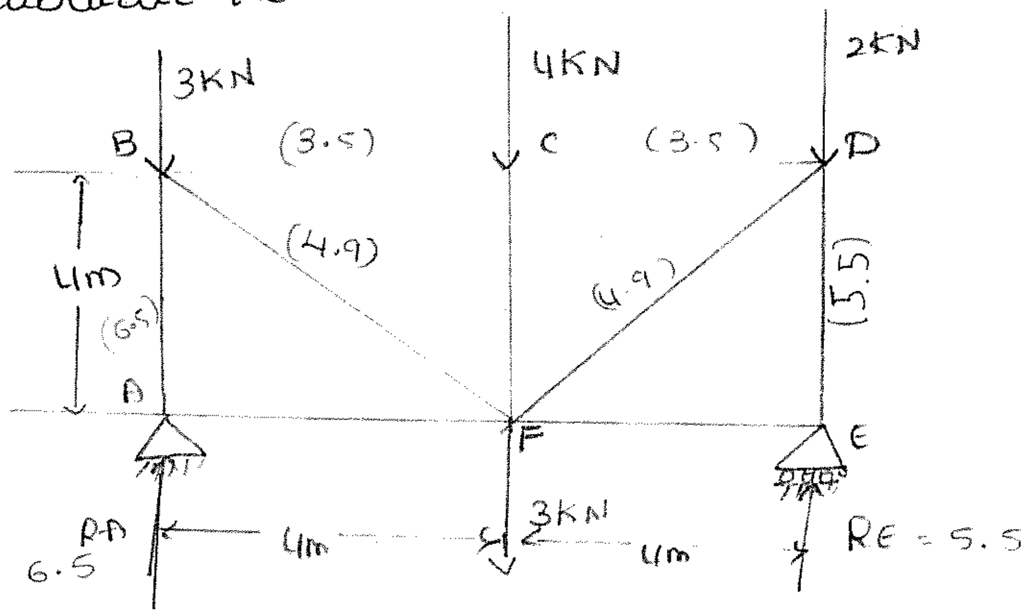
Check $= \Sigma H = 0$

$60.62 - 31.76 \cos 60 - 89.49 \cos 60 = -5 \times 10^{-3} \approx 0$

Hence analysis is Correct.

Member	Force in KN	Nature
AB	83.72	C
BC	60.62	C
CD	89.49	C
AE	41.86	T
ED	44.75	T
BE	37.73	T
CE	31.75	T

3] Analysis the truss shown in the figure and tabulate the results.



$\Sigma V = 0$

$R_A + R_E = 3+4+2+3 = 12$

$\Sigma M_A = 0$

$R_E \times 8 = 4\times4 + 3\times4 + 2\times8$

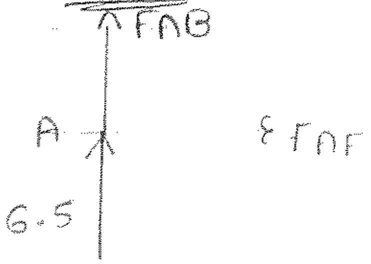
$$R_E = \frac{16+12+16}{8}$$

$R_E = 5.5 \underline{\underline{KN}}$

$R_A + R_E = 12$

$R_A = 12 - 5.5$

Joint A



$\Sigma V = 0$

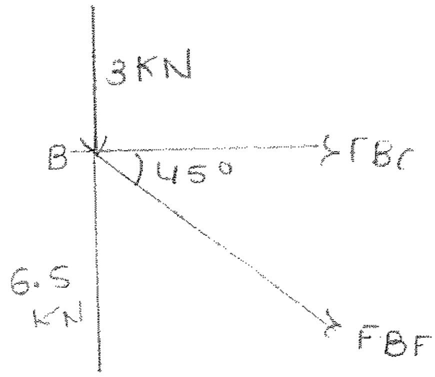
$6.5 + F_{AB} = 0$

$F_{AB} = -6.5\ kN\ (C)$

$\Sigma H = 0$

$F_{AF} = 0$

Joint B.



$\Sigma V = 0$

$-3 + 6.5 - F_{BF} \sin 45 = 0$

$F_{BF} = \dfrac{3.5}{\sin 45}$

$F_{BF} = 4.9\ kN\ (T)$

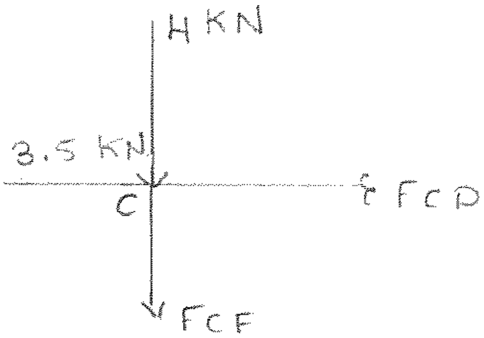
$\Sigma H = 0$

$F_{BC} + F_{BF} \cos 45 = 0$

$F_{BC} = -4.9 \sin 45$

$F_{BC} = -3.5\ kN\ (C).$

Joint C



$\Sigma V = 0$

$-4 - F_{CF} = 0$

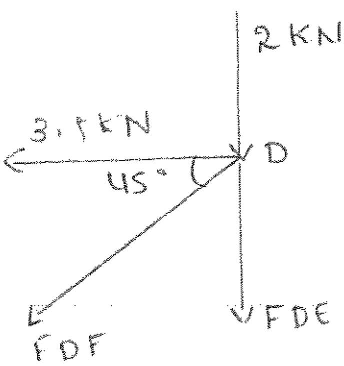
$F_{CF} = -4\ kN\ (C)$

$\Sigma H = 0$

$F_{CD} + 3.5 = 0$

$F_{CD} = -3.5\ kN\ (C)$

Joint D



$\Sigma V = 0$

$-2 - F_{DE} - F_{DF} \sin 45 = 0$

$2 + F_{DF} \sin 45 = -F_{DE} \rightarrow (1)$

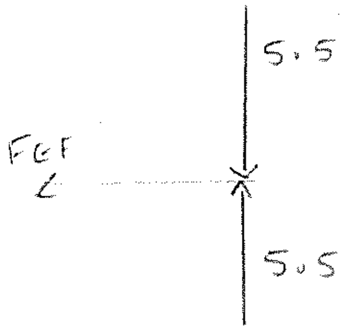
$\Sigma H = 0$

$3.5 - F_{DF} \cos 45 = 0$

$$F_{DF} = \frac{3.5}{\cos 45} = 4.9\,KN\,(T)$$

$$F_{DE} = -(2 + 4.9\sin 45) = -5.5\,KN\,(C)$$

Joint E



$\Sigma V = 0$

$\Sigma H = 0$

$F_{EF} = 0$

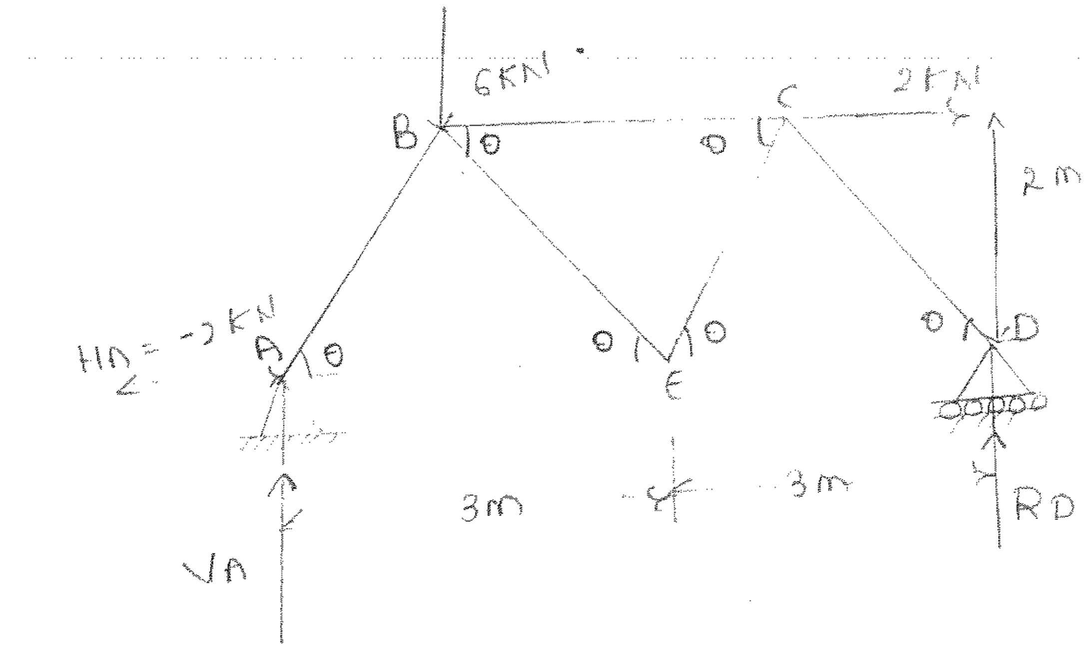
Check $\Sigma V = 0$

$5.5 - 5.5 = 0.$

Hence analysis is Correct.

Member	Force in KN	Nature
AB	6.5	C
BC	3.5	C
CD	3.5	C
DE	5.5	C
AF	0	-
FE	0	-
BF	4.9	T
CF	4	C
DF	4.9	T

4] Determine forces in all members of the truss shown in the figure. Tabulate the results.



To find the reactions,

$\Sigma H = 0$

$H_A + 2 = 0$

$H_A = -2KN$

$\therefore H_A = 2KN$

$\Sigma V = 0$

$V_A + R_D = 6$

$\Sigma M_A = 0$

$(R_D \times 6) = (6 \times 1.5) + (2 \times 2)$

$R_D = 2.16 KN$

$V_A + R_D = 6$

$V_A = 6 - 2.16$

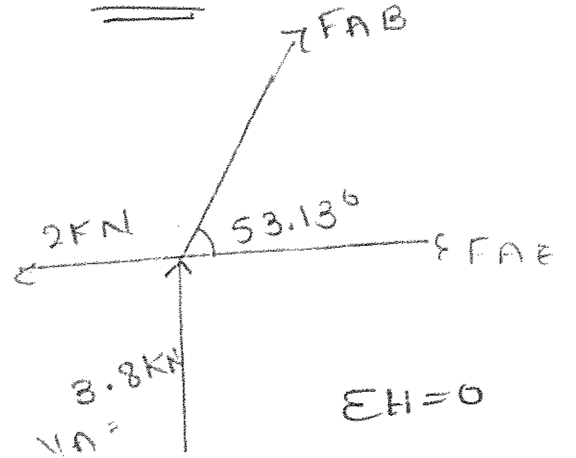
$V_A = 3.84 KN$

$\tan\theta = \frac{2}{1.5}$

$\theta = \tan^{-1}\left(\frac{2}{1.5}\right)$

$\theta = 53.13^\circ$

Joint A



$\Sigma V = 0$

$F_{AB} \sin 53.13 + 3.84 = 0$

$F_{AB} = \frac{-3.84}{\sin 53.13}$

$F_{AB} = -4.8\ (C)$

$\Sigma H = 0$

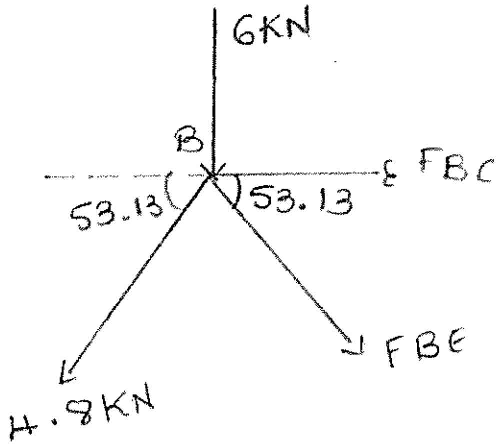
$-2 + F_{AE} + F_{AB} \cos 53.13 = 0$

$F_{AE} = 2 - F_{AB} \cos 53.13$

$F_{AE} = 2 + 4.8 \cos 53.13$

$F_{AE} = 4.88 \text{ KN } (T)$

Joint B



$\Sigma V = 0$

$-6 + 4.8 \sin 53.13 - F_{BE} \sin 53.13$

$-F_{BE} \sin 53.13 = 6 - 4.8 \sin 53.13$

$-F_{BE} = \dfrac{6 - 4.8 \sin 53.13}{\sin 53.13}$

$F_{BE} = -2.7 \text{ KN } (C)$

$\Sigma H = 0$

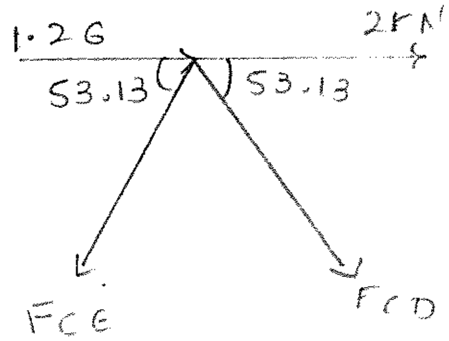
$F_{BC} + F_{BE} \cos 53.13 + 4.8 \cos 53.13 = 0$

$F_{BC} = -F_{BE} \cos 53.13 - 4.8 \cos 53.13$

$F_{BC} = 2.7 \cos 53.13 - 4.8 \cos 53.13$

$F_{BC} = -1.26 \text{ KN } (C)$

Joint c



$\Sigma V = 0$

$-F_{CE} \sin 53.13 - F_{CD} \sin 53.13 = 0 \quad \rightarrow (1)$

$\Sigma H = 0$

$2 + 1.26 + F_{CD} \cos 53.13 - F_{EC} \cos 53.13 = 0$

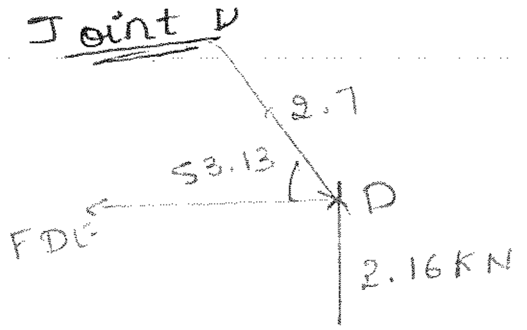
$-F_{EC} \cos 53.13 + F_{CD} \cos 53.13 = -3.26$

$F_{CD} = \dfrac{-3.26 + F_{EE} \cos 53.13}{\cos 53.13} \quad \rightarrow (2)$

By solving eq^{ns} we get

$F_{CE} = 2.7 \ (T) \qquad F_{CD} = -2.7 \ (C)$

Joint D



$\Sigma V = 0$

$2.16 - 2.7 \sin 53.13 = 0$

$2.16 - 2.159 = 0$

≈ 0

$\therefore$ Hence analysis is correct

$\Sigma H = 0$

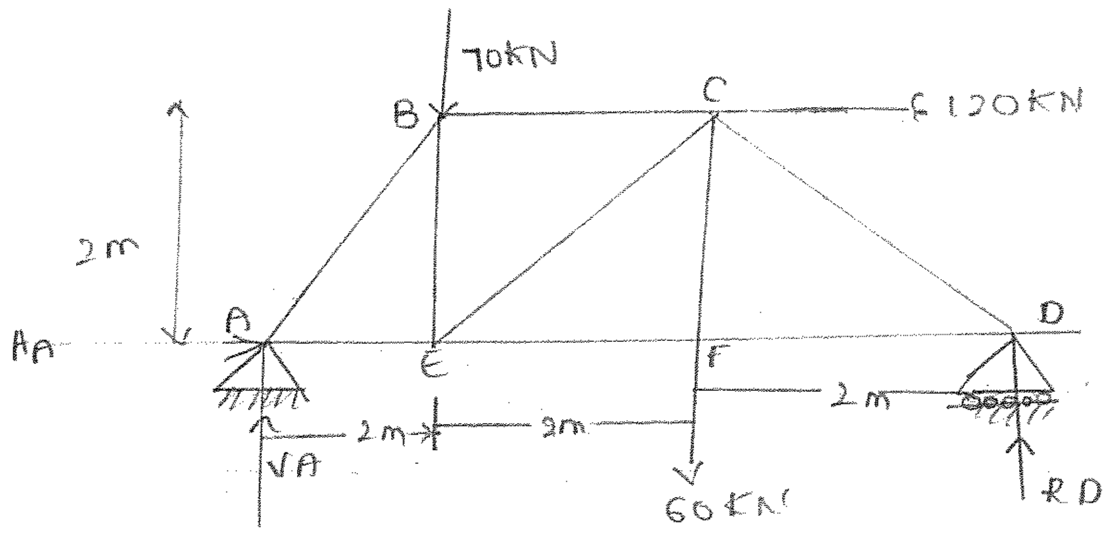
$2.7 \cos 53.13 - FDE = 0$

$FDE = 1.66$ KN (T)

Member	Force in KN	Nature
AB	4.8	C
BC	1.26	C
CD	2.7	C
AE	4.9	T
ED	1.66	T
EB	2.7	C
EC	2.7	T

5] a. Explain briefly the classification of trusses with examples.

b. Determine forces in all the members of the truss shown in the figure. Tabulate the results.



$\Sigma V = 0$

$V_A + R_D = 70 + 60$ $\Rightarrow$ $V_A = 130 - 103.33$

$V_A + R_D = 130$ $\quad$ $\underline{V_A = 26.27\ KN}$

$\Sigma H = 0$

$H_A + 120 = 0$

$H_A = -120 KN$

$\underline{H_A = 120 KN}$

$\Sigma M_A = 0$

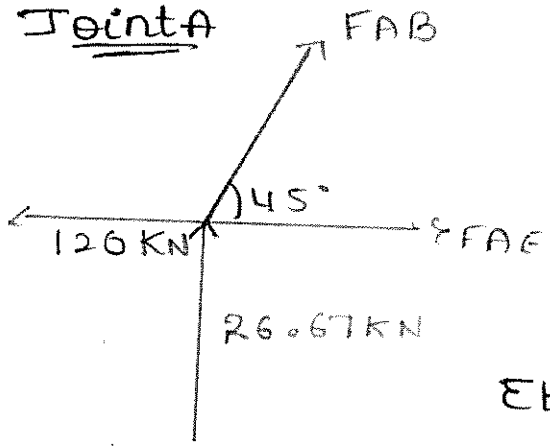
$R_D \times 6 = 70 \times 2 + 60 \times 4 + 120 \times 2$

$$R_D = \frac{140 + 240 + 240}{6}$$

$$R_D = \frac{620}{6}$$

$\underline{R_D = 103.33 KN}$

Joint A



$\Sigma V = 0$

$26.67 + F_{AB} \sin 45 = 0$

$$F_{AB} = \frac{-26.67}{\sin 45}$$

$\underline{F_{AB} = -37.72\ KN\ (C)}$

$\Sigma H = 0$

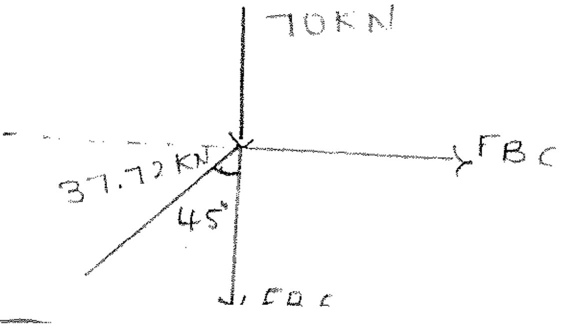
$-120 + F_{AE} + F_{AB} \cos 45 = 0$

$F_{AE} = 120 - (-37.72) \cos 45$

$F_{AE} = 120 + 37.72 \cos 45$

$F_{AE} = 146.67\ (T)$

Joint B



$\Sigma V = 0$

$-70 - F_{BE} \cancel{\sin 45} + 37.72 \sin 45 = 0$

$-F_{BE} = 70 - 37.72 \sin 45$

$F_{BE} = -43.33\ (C)$

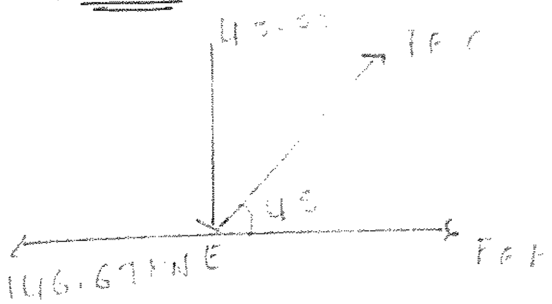
$\Sigma H = 0$

$37.72 \cos 45 + F_{BC} = 0$

$F_{BC} = -37.72 \cos 45$

$F_{BC} = -26.67 (C)$

Joint E



$\Sigma V = 0$

$-43.33 + F_{EC} \sin 45 = 0$

$F_{EC} = \dfrac{43.33}{\sin 45}$

$F_{EC} = 61.28\ KN\ (T)$

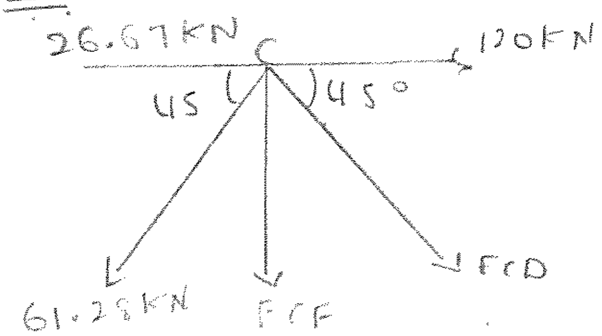
$\Sigma H = 0$

$-146.67 + F_{EF} + F_{EC} \cos 45 = 0$

$F_{EF} = 146.67 - 61.28 \cos 45$

$F_{EF} = 103.34\ KN\ (T).$

Joint C



$\Sigma V = 0$

$-F_{CF} - 61.28 \sin 45 - F_{CD} \sin 45 = 0$

$-F_{CF} - F_{CD} \sin 45 = 61.28 \sin 45 \quad \text{---} (1)$

$\Sigma H = 0$

$26.67 + 120 - 61.28 \cos 45 + F_{CD} \cos 45 = 0$

$F_{CD} = \dfrac{-26.67 - 120 + 61.28 \cos 45}{\cos 45}$

$F_{CD} = -146.14\ (C).$

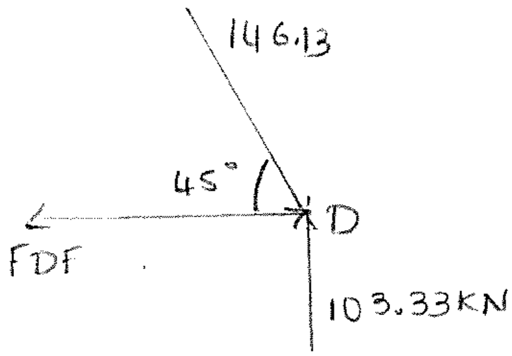
F_{CD} substituting in (1) we get F_{CF}

$-F_{CF} - (-146.14) \sin 45 = 61.28 \sin 45$

$-F_{CF} = 61.28 \sin 45 - 146.14 \sin 45$

$F_{CF} = 60\ KN\ (T)$

Joint D



$\Sigma H = 0$

$-FDF + 146.13 \cos 45 = 0$

$-FDF = -146.13 \cos 45$

$FDF = 103.33\ KN\ (T)$

$\Sigma V = 0$

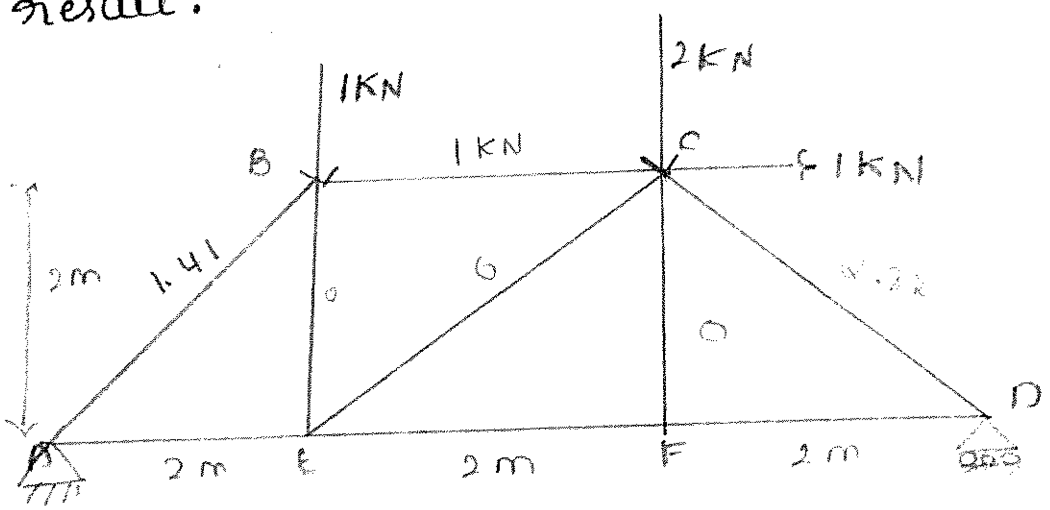
$103.33 - 146.13 \cos 45$

$= \underline{\underline{0}}.$

Hence analysis is correct.

Member	Force in KN	Nature
AB	37.72	C
BC	26.67	C
CD	146.13	C
AE	146.67	T
EF	103.34	T
FD	103.34	T
BE	43.33	C
EC	61.28	T
CF	60	T

6] Analyse the truss shown in figure and tabulate the result.



$\Sigma V = 0$

$V_A + R_D = 1 + 2$

$V_A + R_D = 3$

$\Sigma H = 0$

$H_A + 1 = 0$

$H_A = -1 \quad \therefore H_A - 1KN \ (\leftarrow)$

$\Sigma M_A = 0$

$R_D \times 6 = 1 \times 2 + 2 \times 4 + 1 \times 2$

$R_D = \dfrac{2 + 8 + 2}{6}$

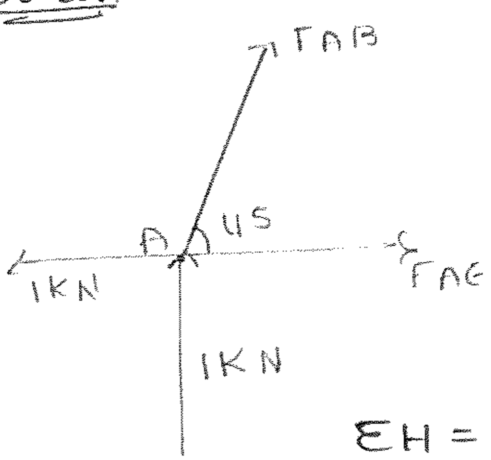
$R_D = \underline{\underline{2KN}}$

$V_A + R_D = 3$

$V_A = 3 - 2$

$V_A = \underline{\underline{1KN}}$

Joint A



$\Sigma V = 0$

$1 + F_{AB} \sin 45 = 0$

$F_{AB} = \dfrac{-1}{\sin 45}$

$F_{AB} = -1.41 \ \underline{\underline{KN}} (C)$

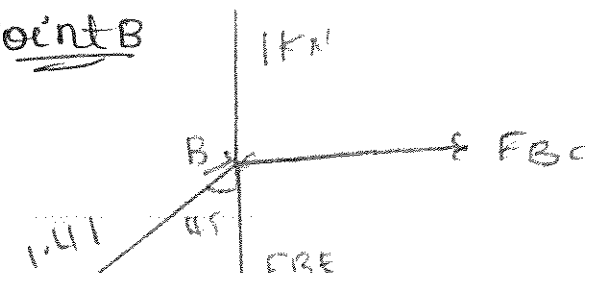
$\Sigma H = 0$

$F_{AE} - 1 + F_{AB} \cos 45 = 0$

$F_{AE} = 1 - (-1.41 \cos 45)$

$F_{AE} = 1.99 \approx 2 \ KN \ (T)$

Joint B



$\Sigma V = 0$

$-1 - F_{BE} + 1.41 \sin 45 = 0$

$F_{BE} = -2.9 \times 10^{-3} \approx 0$

$F_{BE} = 0$

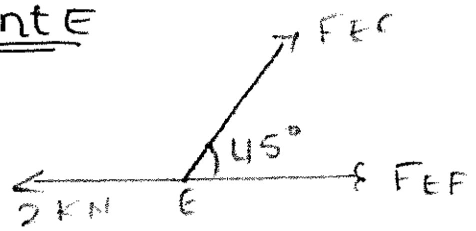
$\Sigma H = 0$

$F_{BC} + 1.41 \cos 45 = 0$

$F_{BC} = -1.41 \cos 45$

$F_{BC} = -0.99 \approx -1\ kN\ (C)$

Joint E



$\Sigma V = 0$

$F_{EC} \sin 45 = 0$

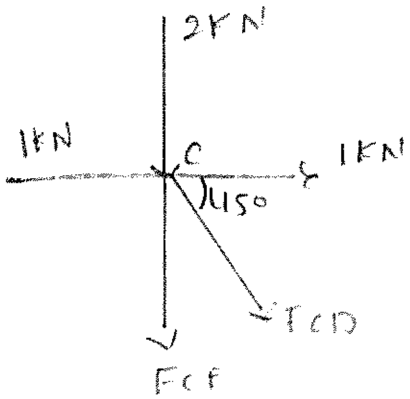
$F_{EC} = 0$

$\Sigma H = 0$

$-2 + F_{EF} = 0$

$F_{EF} = 2\ (T)$

Joint C



$\Sigma V = 0$

$-2 - F_{CF} - F_{CD} \sin 45 = 0$

$-F_{CF} = 2 + F_{CD} \sin 45 \rightarrow (1)$

$\Sigma H = 0$

$1 + 1 + F_{CD} \cos 45 = 0$

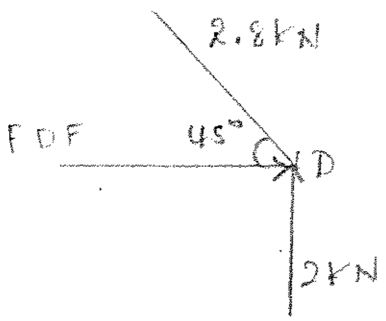
$F_{CD} = \dfrac{-2}{\cos 45}$

$F_{CD} = -2.8\ KN\ (C)$

$-F_{CF} = 2 - 2.8 \sin 45$

$F_{CF} = -0.02 \approx 0$

Joint D



$\Sigma H = 0$

$-F_{DF} + 2.8 \cos 45 = 0$

$F_{DF} = 2.8 \cos 45°$

$F_{DF} = 1.97 \approx 2$

$F_{DF} = 2$

$\Sigma V = 0$

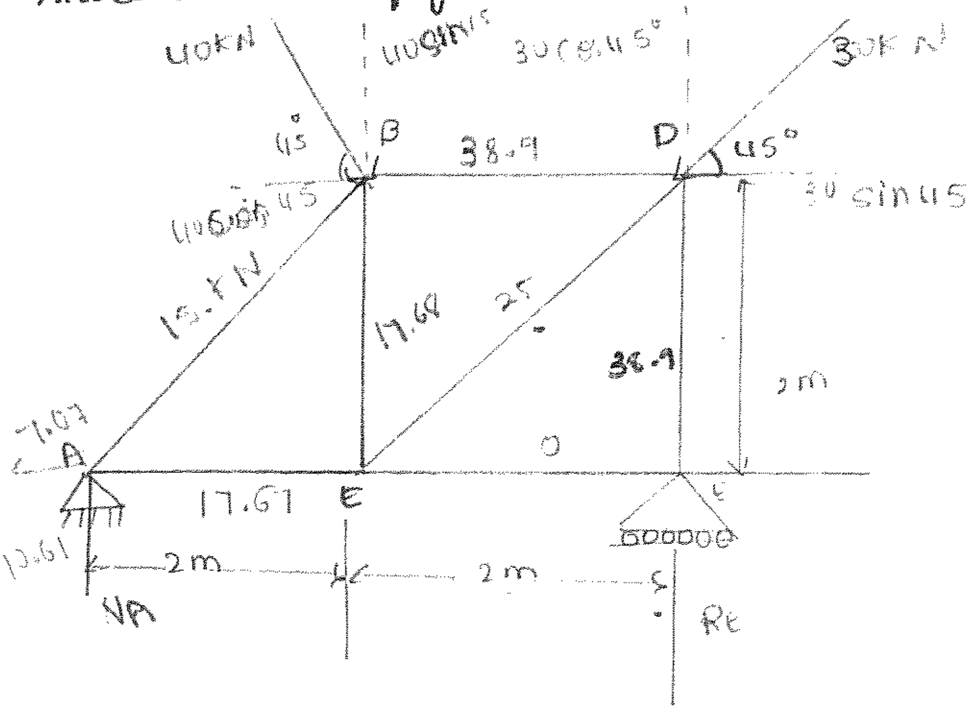
$\Sigma V = 2 - 2.8 \sin 45 = 0$

$\Sigma V = 2 - 1.97 \approx 0$

Hence analysis is correct.

Member	Force in KN	Nature
AB	1.41	C
BC	1	C
CD	2.82	C
AE	2	T
EF	2	T
FD	2	T
EB	0	—
EC	0	—
FC	0	—

7] Determine forces in all the members of the truss shown in the figure and Tabulate the results.



$\Sigma V = 0$

$RA + RE = 30 + 40$

$RA + RE = 70$

$\Sigma H = 0$

$H_A + 40\cos 45° - 30\cos 45° = 0$

$H_A = 30\cos 45 - 40\cos 45$

$\Sigma M_A = 0 \quad H_A = -7.07 \; (\leftarrow)$

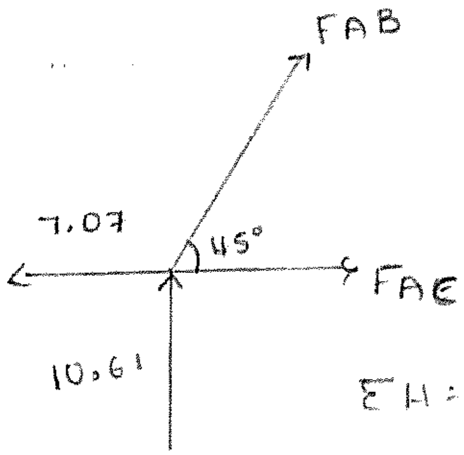
$(R_E \times 4) = 40\sin 45 \times 2 + 30\sin 45 \times 4 + 40\cos 45 \times 2 - 30\cos 45 \times 2$

$R_E = 38.89 \, KN$

$V_A + R_E =$

$V_A = 10.61$

Joint A



$\Sigma V = 0$

$10.61 + F_{AB} \sin 45 = 0$

$F_{AB} = -\dfrac{10.61}{\sin 45}$

$F_{AB} = -15.0 \, KN \; (C)$

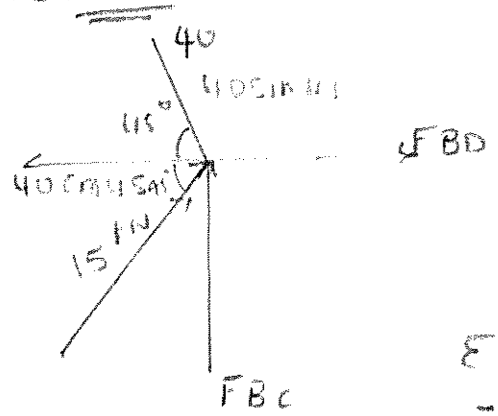
$\Sigma H = 0$

$-7.07 + F_{AC} + F_{AB} \cos 45 = 0$

$F_{AC} = 7.07 + 15(\cos 45) = 0$

$F_{AC} = 17.67 \, KN \; (T)$

Joint B



$\Sigma V = 0$

$+40\cos 45° + F_{BD} + 15\cos 45° = 0$

$F_{BD} = -15\cos 45° - 40\cos 45°$

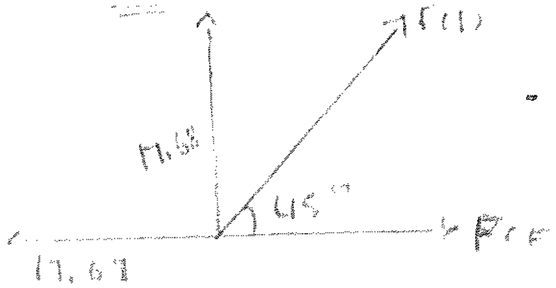
$F_{BD} = -38.9$

$\Sigma H = 0$

$-40\sin 45° + F_{BC}\sin 90 + 15\sin 45 = 0$

$F_{BC} = -17.68 \, KN$

Joint C



$\Sigma V = 0$

$F_{CD} \sin 45° - 17.68 = 0$

$F_{CD} = \dfrac{17.68}{\sin 45°}$

$F_{CD} = 25$ kN

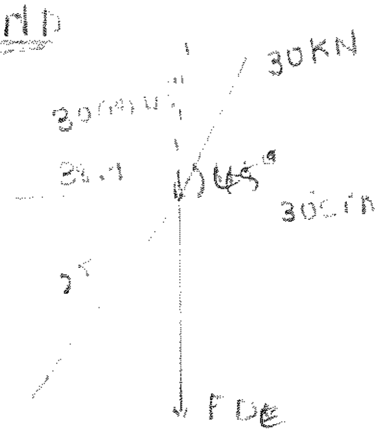
$\Sigma H = 0$

$F_{CD} \cos 45° - 17.68 + F_{CE}$

$F_{CE} = -25 \cos 45° + 17.68$

$F_{CE} = 0.$

Joint D



$\Sigma H = 0$

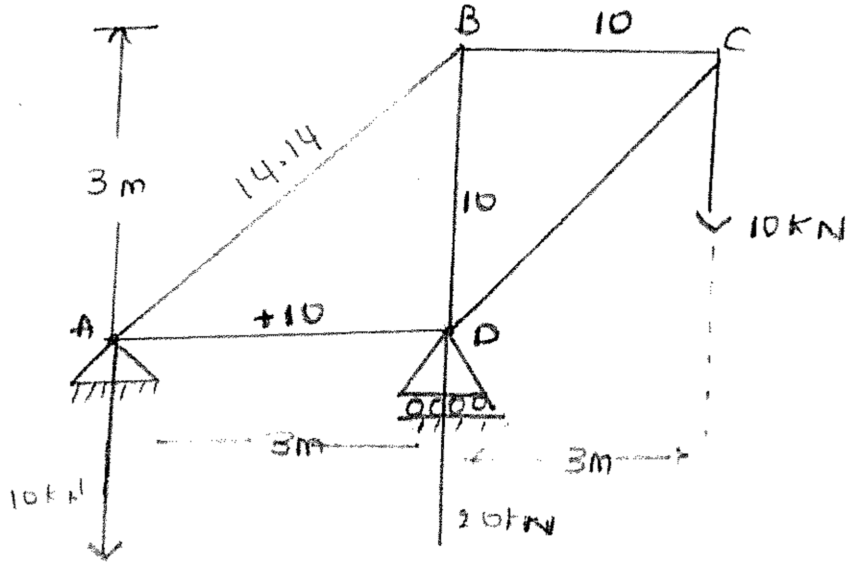
$38.9 + F_{DE}$

$F_{DE} = -38.9$

$\Sigma V = 0.$

Member	Force in kN	Nature
AB	15	C
BC	17.68	C
BD	38.9	C
DE	38.9	C
AC	17.68	T
CE	0	-
CD	25	T

8] Analyse the truss [frame as shown in the figure and tabulate the result.



$\Sigma V = 0$

$\cancel{R}A + R_D = 10$

$\Sigma M_A =$

$(R_D \times 3) = (10 \times 6)$

$R_D = \frac{\cancel{60}^{20}}{\cancel{3}}$

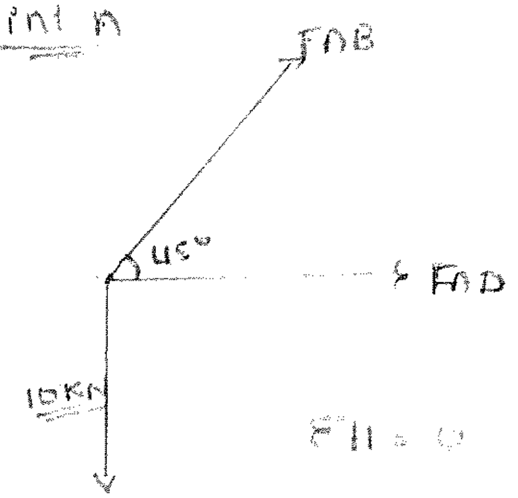
$\boxed{R_D = 20}$

$V_A + R_D = 10$

$V_A = 10 - 20$

$\boxed{V_A = -10\,KN}$

Joint A



$\Sigma V = 0$

$F_{AB} \sin 45° - 10 = 0$

$F_{AB} = \frac{10}{\sin 45°}$

$F_{AB} = 14.14\ KN\ (T)$

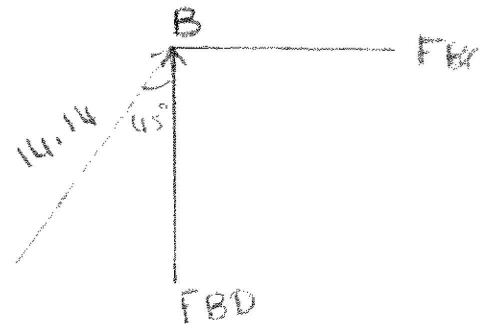
$\Sigma H = 0$

$F_{AD} + F_{AB} \cos 45$

$F_{AD} = -14.14 \cos 45$

$F_{AD} = -10\ KN\ (C)$

Joint B



$\Sigma V = 0$

$14.14 \sin 45 + F_{BC} = 0$

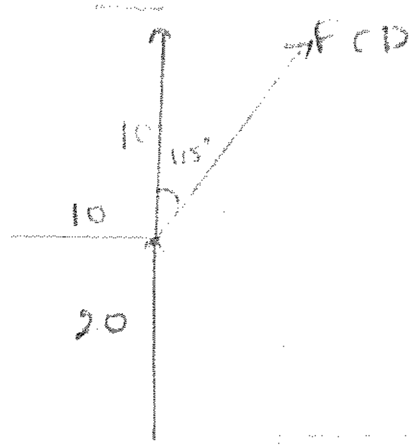
$F_{BC} = -10\ kN\ (C)$

$\Sigma H = 0$

$F_{BD} - 14.14 \cos 45 = 0$

$F_{BD} = 10\ kN$

Joint D



$\Sigma V = 0$

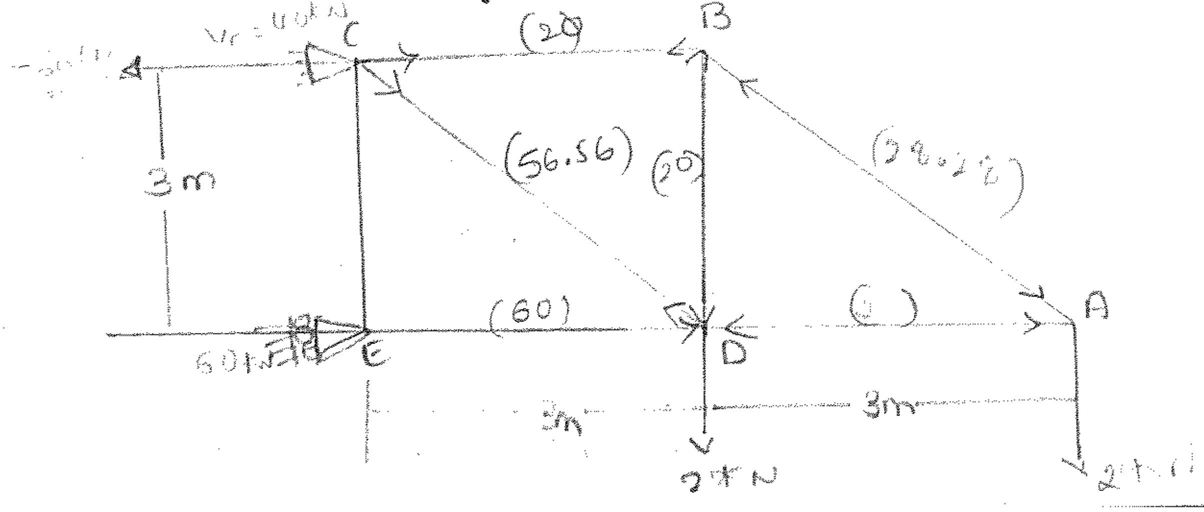
$F_{CD} \sin 45° + 20 = 0$

$F_{CD} = \frac{-20}{\sin 45}$

$F_{CD} = 14.14\ kN$

Member	Force in kN	Nature
AB	14.14	T
BC	10	T
AD	10	C
DC	14.14	C
BD	10	C

9] Determine forces all the members of cantilever truss shown in the figure. and tabulate the results.



$\Sigma H = 0$

$R_E + H_C = 0$

$H_C = -\underline{R_E}$

$\Sigma V = 0$

$V_C = 20 + 20$

$\boxed{V_C = 40 KN}$

$\Sigma M_A = 0$

$(R_E \times 3) = (20 \times 3) + (20 \times 6)$

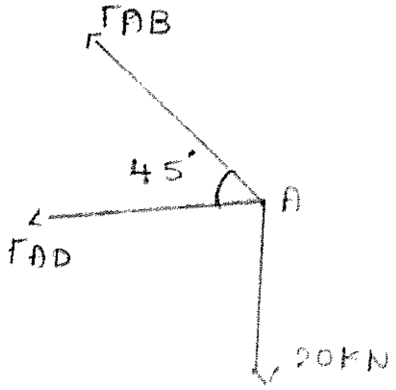
$R_E = \frac{60 + 120}{3}$

$\boxed{R_E = +60 KN}$

But $H_C = -R_E$

$\boxed{H_C = -60 KN}$

Joint A



$\Sigma V = 0$

$F_{AB} \sin 45° - 20 = 0$

$F_{AB} = \frac{20}{\sin 45}$

$F_{AB} = 28.28 KN (T)$

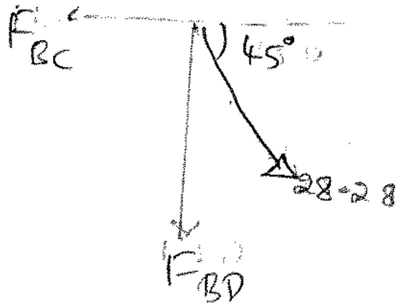
$\Sigma H = 0$

$-F_{AB} \cos 45° - F_{AD} = 0$

$-F_{AD} = 28.28 \cos 45°$

$F_{AD} = -\underline{20} (C)$

Joint B



$\Sigma V = 0$

$-28.28 \sin 45° - F_{BD} = 0$

$F_{BD} = -28.28 \sin 45°$

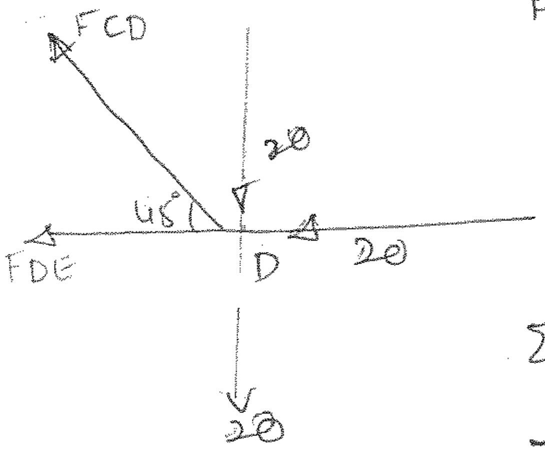
$F_{BD} = -20 (C)$

$\Sigma H = 0$

$-F_{BC} + 28.28 \cos 45° = 0$

$+F_{BC} = +20 (T)$

Joint - D



$\Sigma V = 0$

$F_{CD} \sin 45° - 20 - 20 = 0$

$F_{CD} = \dfrac{40}{\sin 45}$

$F_{CD} = 56.56 \ (T)$

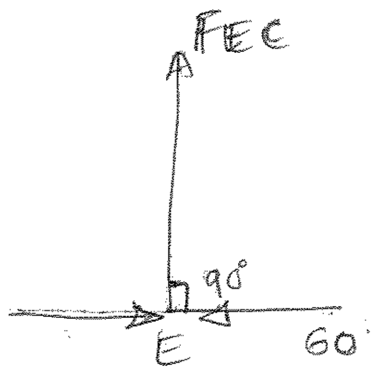
$\Sigma H = 0$

$-F_{DE} - F_{CD} \cos 45° - 20 = 0$

$-F_{DE} = 20 + 56.56 \cos 45$

$F_{DE} = -60 \ (C)$

Joint - E



$\Sigma V = 0$

$F_{EC} = 0$

$\Sigma H = 0$

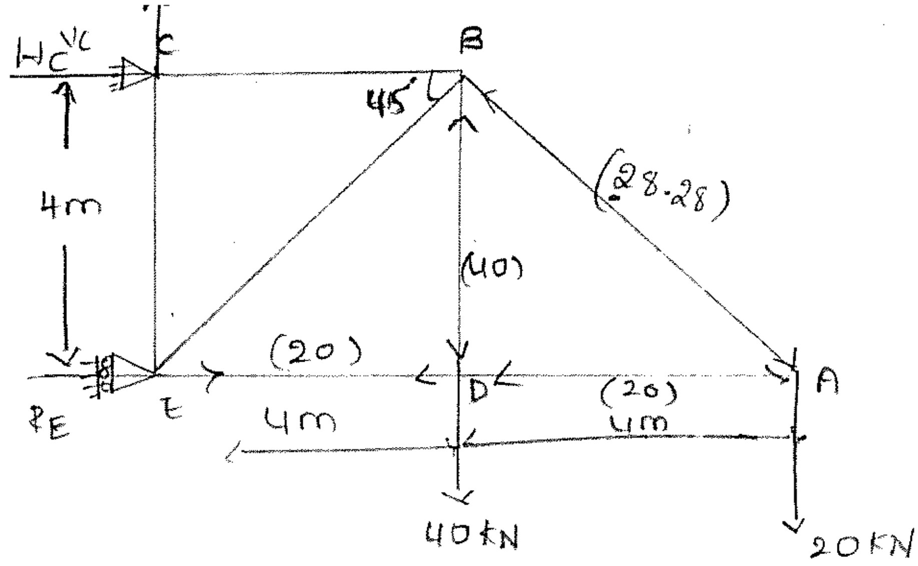
$60 - 60 = 0$

$0 = 0$

Hence anlaysis is correct

Member	Force in KN	Nature
AB	28.28	T
AD	-20	C
BD	-20	C
BC	+20	T
CD	56.56	T
DE	-60	C
EC	0	—

10.



$\Sigma V = 0$

$V_C - 40 - 20 = 0$

$V_C = 60\ KN\ (\uparrow)$

$\Sigma H = 0$

$H_C + R_E = 0$

$\Sigma M_A = 0$

$40 \times 4 + 20 \times 8 - R_E \times 4 = 0$

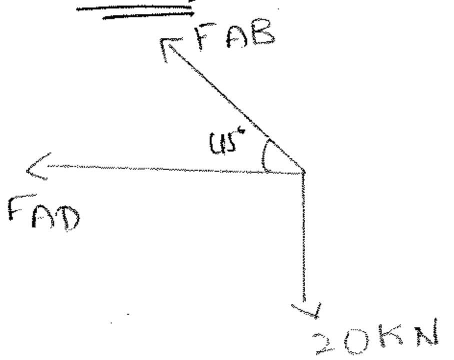
$R_E = \frac{160 + 160}{4} = \frac{320}{4}$

$\boxed{R_E = 80 KN}$

$H_C + R_E = 0$

$H_C = \underline{\underline{-80 KN}}\ (\leftarrow)$

Joint A



$\Sigma V = 0$

$-20 + F_{AB} \sin 45° = 0$

$F_{AB} = \frac{20}{\sin 45°}$

$F_{AB} = \underline{\underline{28.28}}\ KN\ (T)$

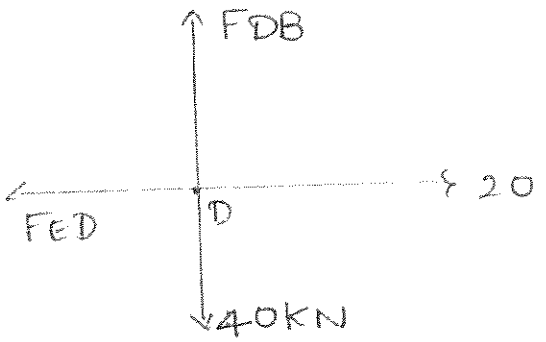
$\Sigma H = 0$

$-F_{AD} - F_{AB}\cos 45^\circ = 0$

$-F_{AD} = 28.28\cos 45^\circ$

$F_{AD} = -19.99\ (C) \approx 20\ KN$

Joint D :-



$\Sigma V = 0$

$F_{DB} = -40KN(C)$

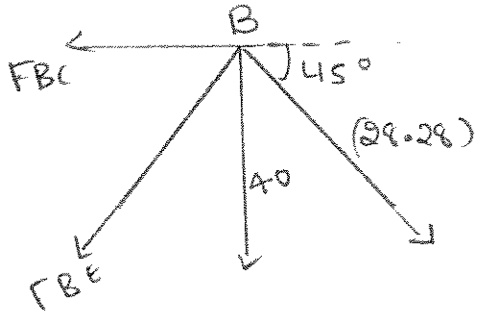
$\Sigma H = 0$

$-F_{ED} - 20 = 0$

$-F_{ED} = 20$

$F_{ED} = -20\ KN(C)$

Joint B:-



$\Sigma V = 0$

$-F_{BE}\sin 45 - 40 - 28.28\sin 45^\circ = 0$

$-F_{BE} = \dfrac{28.28\sin 45^\circ + 40}{\sin 45^\circ}$

$-F_{BE} = 28.28\sin 45 + 40$

$F_B = -84.85\ KN\ (C)$

$-F_{BE} = \dfrac{20.140}{\sin 45^\circ} = -20$

$F_{BE} = -28.28\ KN(C)$

$\Sigma H = 0$

$-F_{BE} - F_{BE}\cos 45^\circ + 28.28\cos 45^\circ = 0$

~~$-F_{BE}\cos 45^\circ = 28.28\cos 45^\circ$~~

$-F_{BE}$

$-F_{BC} = -28.28\cos 45^\circ + (-28.28)\cos 45^\circ$

$F_{BC} = 39.9 \approx 40\ KN\ (T)$

=0

-40

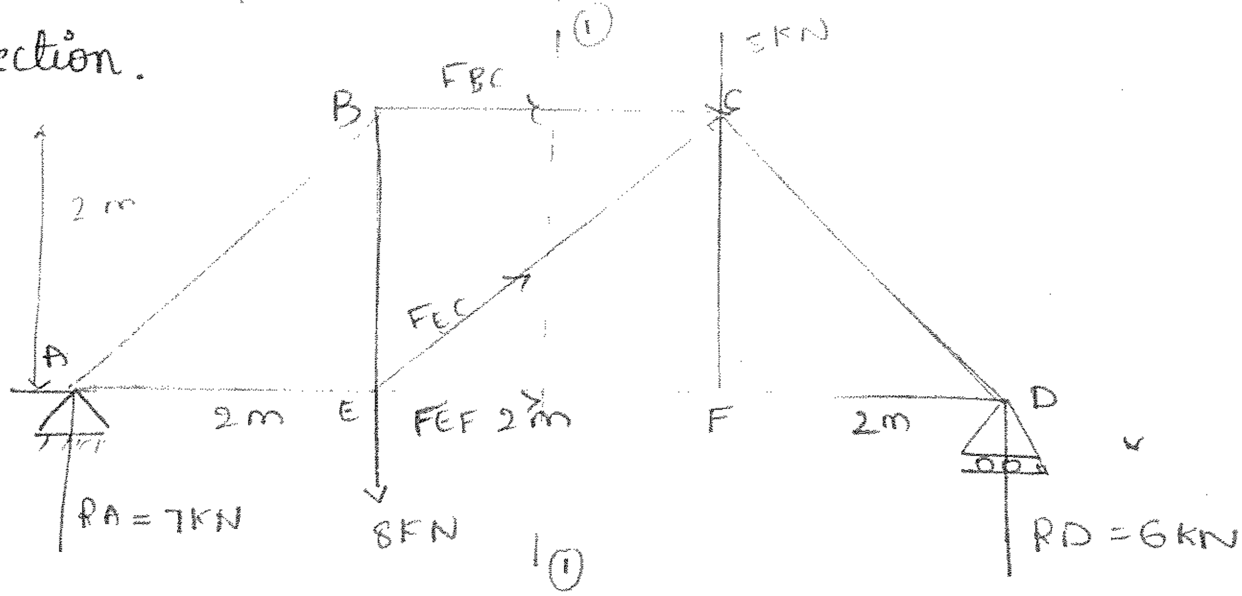
8)

5°

T))

Analysis of Trusses by the method of ~~trusses~~ Section :-

1] Determine the forces in members BC, EF and EC of the truss shown in the figure by the method of section.



$\Sigma V = 0$

$R_A + R_D = 8 + 5$

$R_A + R_D = \underline{13\ KN}$

$R_A = 13 - 6$

$R_A = \underline{7KN}$

$\Sigma M_A = 0$

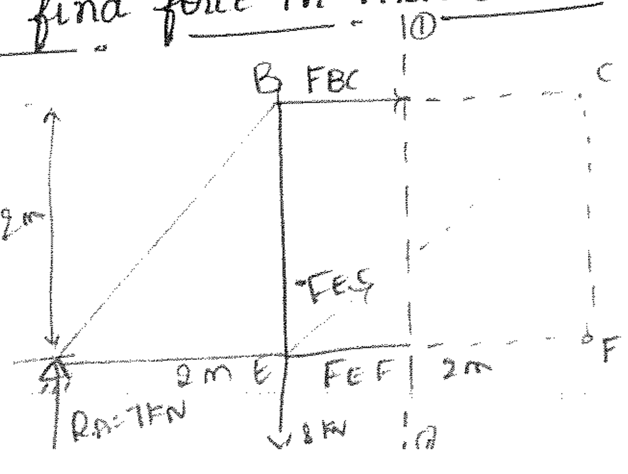
$R_D \times 6 = 8 \times 2 + 5 \times 4$

$R_D = \dfrac{16 + 20}{6}$

$R_D = \underline{6\ KN}$

Consider FBD to the left of 1-1

To find force in member BC



$$\Sigma M_E = 0$$

$$(F_{BC} \times 2) + (7 \times 2) = 0$$

$$F_{BC} = \frac{-14}{2}$$

$$F_{BC} = -7\ KN\ (C)$$

To find force in member EF

$$\Sigma M_C = 0$$

$$(7 \times 4) - (8 \times 2) - (F_{EF} \times 2) = 0$$

$$F_{EF} = +\frac{12}{2}$$

$$F_{EF} = +6\ KN\ (T)$$

To find force in member EC

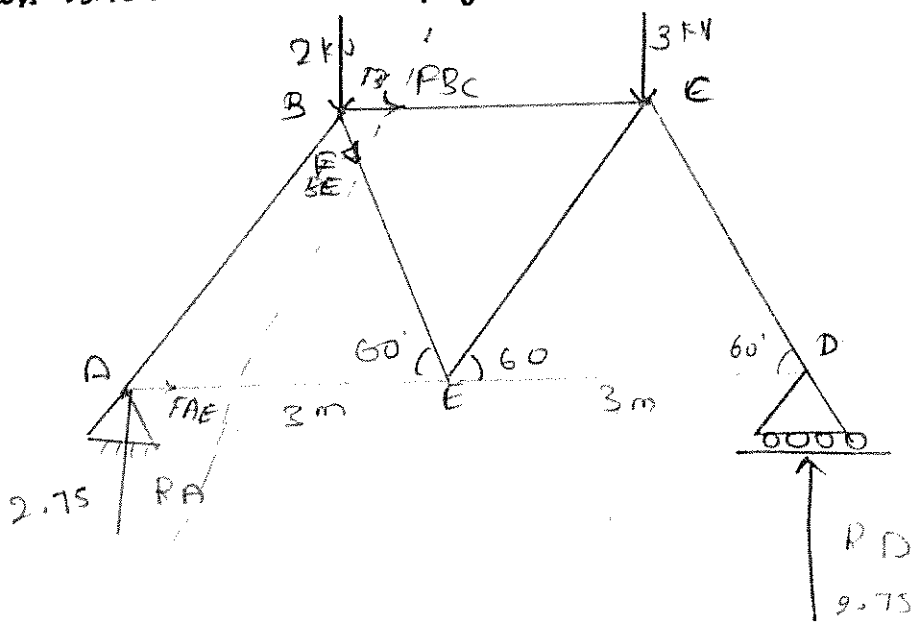
$$\Sigma Y = 0$$

$$7 - 8 + F_{EC} \sin 45 = 0$$

$$F_{EC} = \frac{1}{\sin 45}$$

$$F_{EC} = 1.41\ KN\ (T)$$

2] Determine the forces in members BC, BE and AE of the truss shown in the figure.



$\Sigma V = 0$

$R_A + R_D = 2 + 3$

$R_A + R_D = 5$

$R_A = 5 - 2.75$

$R_A = 2.25\ \text{kN}$

$\Sigma M_A = 0$

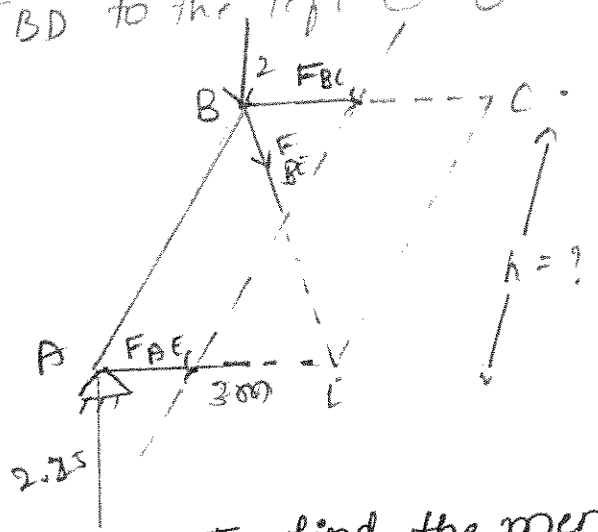
$R_D \times 6 = 2 \times 1.5 + 3 \times 4.5$

$$R_D = \frac{2 \times 1.5 + 3 \times 4.5}{6}$$

$R_D = 2.75\ \text{kN}$

③

FBD to the left ① ①



$\tan 60 = \frac{h}{1.5}$

$h = 2.6$

$\sin 60 = \frac{h}{3}$

$h = 2\text{m}$

To find the member in BC

$\Sigma M_E = 0$

$(2.25 \times 3) + (F_{BC} \times 2.6) - (2 \times 1.5) = 0$

$$F_{BC} = \frac{-(2.25 \times 3) + (2 \times 1.5)}{2.6}$$

$F_{BC} = -1.44\ \text{kN (C)}$

To find the member in AE

$\Sigma M_B = 0$

$(2.25 \times 1.5) - (F_{AE} \times 2.6) = 0$

$$F_{AE} = \frac{2.25 \times 1.5}{2.6}$$

$F_{AE} = 1.3\ \text{kN (T)}$

To find the member in BE

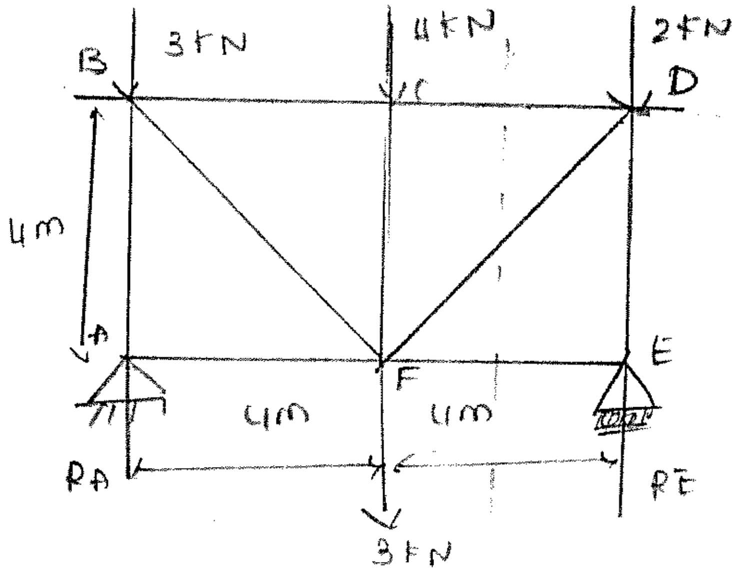
$\Sigma V = 0$

$+2.25 - 2 - F_{BE} \sin 60 = 0$

$F_{BE} = \dfrac{-2.25 + 2}{-\sin 60}$

$F_{BE} = 0.28 \text{ kN } (T)$

(3) DC, DF and EF.



$\Sigma V = 0$

$R_A + R_E = 3 + 3 + 4 + 2$

$R_A + R_E = 12$

$R_A = 12 - 5.5$

$R_A = 6.5 \text{ kN}$

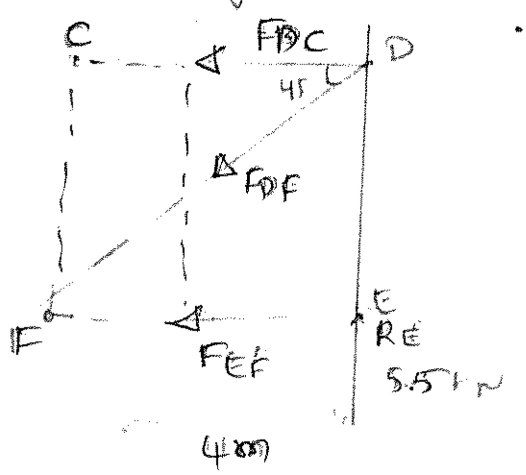
$\Sigma M_A = 0$

$(R_E \times 8) = (4 \times 4) + (2 \times 8) + (3 \times 4)$

$R_E = \dfrac{16 + 16 + 12}{8}$

$R_E = 5.5 \text{ kN}$

FBD to the Right (1)-(1)



To find the member in DC

$\Sigma M_F = 0$

$-(F_{DC} \times 4) - (5.5 \times 4) + (2 \times 4) = 0$

$F_{DC} = -3.5 \text{ kN } (C)$

To find member in EF

$\Sigma M_D = 0$

$F_{EF} \times 4 = 0$

$F_{EF} = 0$

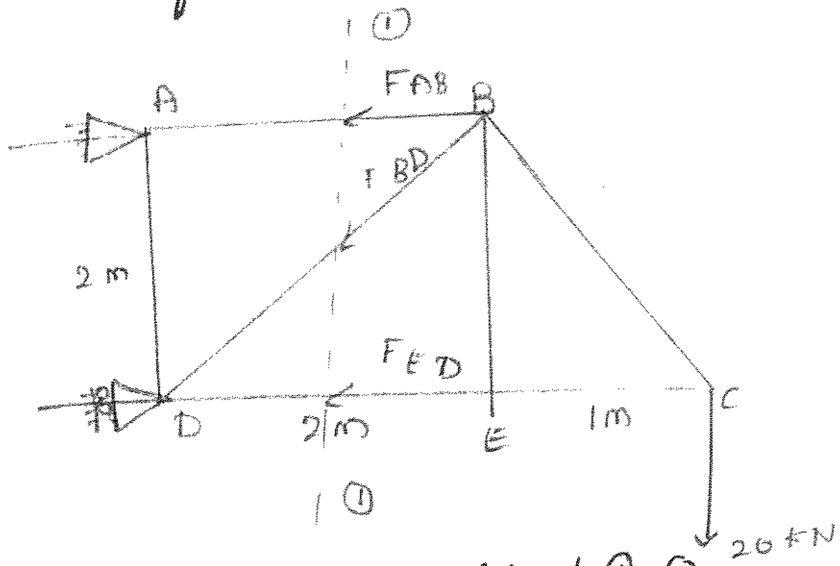
To find the member in DF

$\Sigma V = 0$

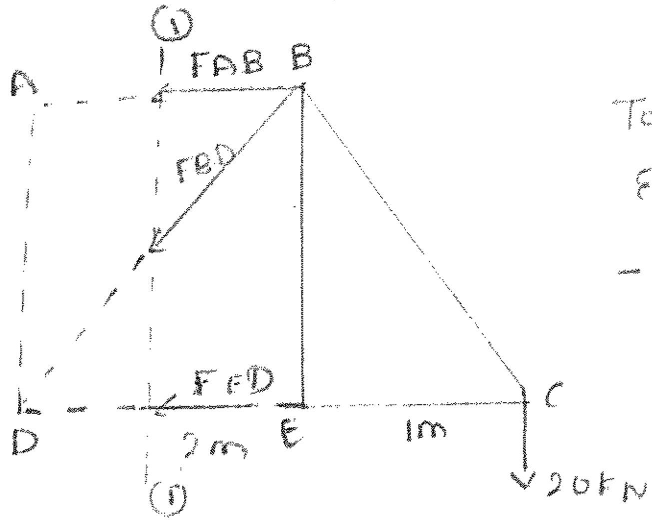
$-2 + 5.5 - F_{DF} \sin 45 = 0$

$F_{DF} = 4.95\ \underline{\underline{kN(T)}}$

Determine forces in the members AB, BE and BD.



Consider FBD to right of (1)-(1)



To find DE

$\Sigma M_B = 0$

$-F_{ED} \times 2 = 20 \times 1$

$F_{ED} = -10\ kN(C)$

To find the member in member AB.

$\Sigma M_D = 0$

$(F_{AB} \times 2) - (20 \times 3) = 0$

$F_{AB} = \frac{60}{2}$

$F_{AB} = 30\ \underline{kN(T)}$

To find BD

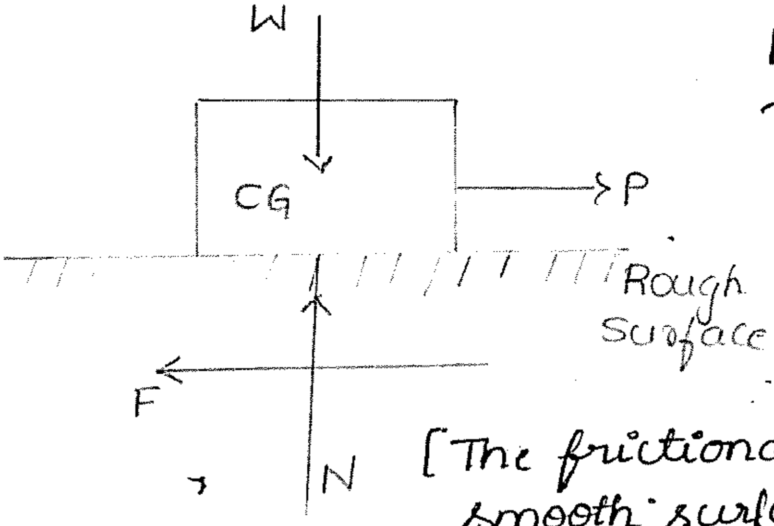
$\Sigma V = 0$

$-20 - F_{BD} \sin 45 = 0$

$F_{BD} = \frac{20}{\sin 45}$

$F_{BD} = \underline{\underline{-28.28}}\ (C)$

FRICTION



W = Weight of the body
P = External force.
F = Frictional force.
N = Normal reaction.

[The frictional force (F) is zero for smooth surfaces]

When a body moves or tends to move over another body a force opposing the motion develops at the contact surfaces. The force which opposes the movem-nt or the tendency of movement is called the friction-al force [or friction].

The resistance offered to motion at the surfaces of the bodies is known as frictional force (F).

Limiting friction :- If the applied tangential force is more than the maximum frictional force there will be movement of one body over the other body.

This maximum value of frictional force, when the motion is impending is known as limiting friction [Fmax]

Types of Friction :-

Static friction :- When the applied force is less than the limiting friction, the body remains at rest and such friction is called static friction.

It is the friction experienced between two bodies when both bodies are at the rest.

Static friction will have any value between zero and limiting friction.

Kinetic Friction / Dynamic friction :-

If the applied force exceeds the limiting frictio the body starts moving over another body and the frictional resistance experienced while moving is known as Dynamic friction. (or) Kinetic friction.

It is the friction experienced between two bodie when one body moves over the other body.

The magnitude of dynamic friction is found to be less than limiting friction.

The dynamic friction may be classified into two groups :-

(i) Sliding friction :- It is the friction experienced by a body when it ~~rolls~~ slide over another body.

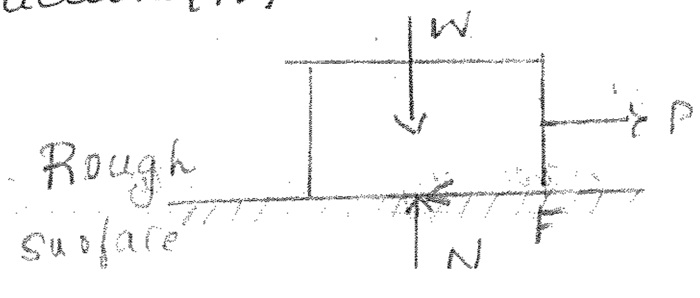
(ii) Rolling friction :- It is the friction experienced by a body when it rolls over another body.

Based on the surfaces of contact, there are two types of friction.

(i) Dry friction :- If the contact surfaces between the two bodies are dry, then the friction between such bodies is known as dry friction.

(ii) Fluid friction :- The friction between two fluid layers or the friction between a solid and a fluid is known as fluid friction.

Co-efficient of friction (μ) :- It is defined as the ratio of the limiting friction (F) to the normal reactions (N) between the two bodies.



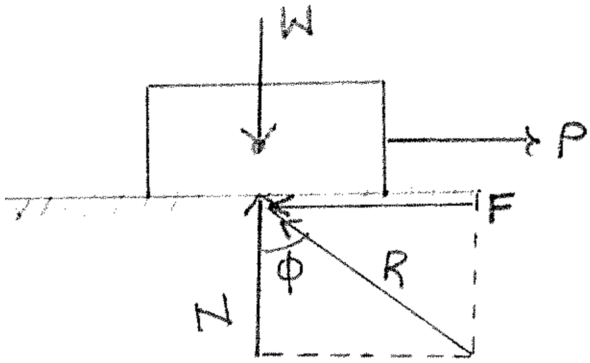
Co-eff of friction = $\dfrac{\text{Limiting friction}}{\text{Normal reactions.}}$

$$\boxed{\mu = \frac{F}{N}}$$

For smooth surface, $\mu = 0$.

Angle of friction (ϕ):

It is defined as the angle made by the resultant (R) of the normal reaction (N) and the force of friction (F) with the normal reaction (N).

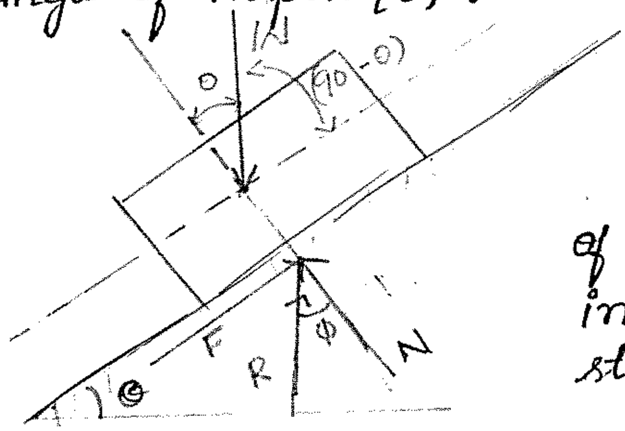


$$\boxed{\tan\phi = \frac{F}{N}}$$

But $\mu = \dfrac{F}{N}$

$$\boxed{\mu = \tan\phi}$$

Angle of respose (θ) :-



Consider a body placed on inclined plane as shown in the figure. Let the angle of inclination (θ) be gradually increased till the body just starts sliding down the plane

The angle of inclined plane at which a body just begins to slide down the plane is called the angle of repose (θ).

To show angle of friction = Angle of respose [i.e $\phi = \theta$] at the limiting equilibrium conditions.

$\Sigma F_{\perp r} = 0$, $N - W\sin(90° - \theta) = 0$ $\therefore N = W\cos\theta$

$\Sigma F_{\parallel} = 0$, $F - W\cos(90° - \theta) = 0$ $\therefore F = W\sin\theta$

We know that, $\mu = \frac{F}{N}$

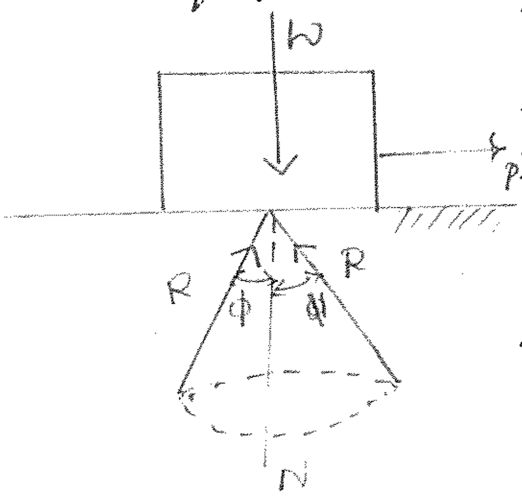
$$\mu = \frac{\cancel{W}\sin\theta}{\cancel{W}\cos\theta} = \tan\theta$$

But $\mu = \tan\phi$ $\therefore$ $\tan\phi = \tan\theta$

$$\underline{\phi = \theta}$$

$\therefore$ Angle of friction = Angle of repose.

Cone of friction :-



The force, P corresponding to the limiting friction as in figure, remains constant in magnitude but revolves in a horizontal plane the force R will move along the surface of a cone as shown in the figure.

This cone is called cone of friction.

Laws of dry friction :-

1] The frictional force always acts in a direction opposite to that in which the body tends to move.

2] The limiting force of friction bears a constant ratio to the normal reaction between the two surfaces i.e $\mu = F/N$.

3] The force of friction is independent of the area of contact between the two surfaces.

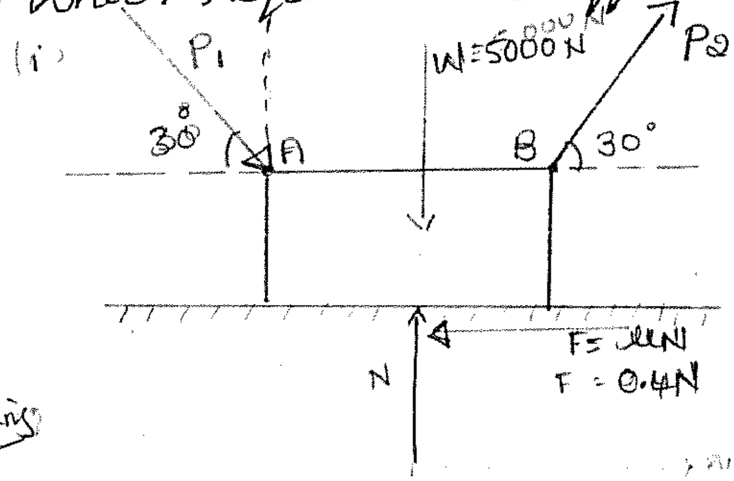
4] The force of friction depends upon the roughness /smoothness of the surfaces.

5] Till the limiting value is reached the magnitude of frictional force is exactly equal to the tangential force which tends to move the body.

6] After the body starts moving the dynamic friction comes into play the magnitude of which is less than that of limiting friction, and it bears a constant ratio to the normal force. This ratio is called coefficient of K.F[illegible]

Problems :-

01] A block of weight 5000 N rests on a horizontal floor and the coeff of friction b/w them as 0.4. The block can be moved by either pushing at A or pulling at B. as shown in the figure. Through calculations show which requires less effort.



(i) pulling

$\Sigma V = 0$

$-P_1 \sin 30^\circ - 5000 + N = 0$

$-P_1 \sin 30 + N = 5000 \rightarrow (1)$

$\Sigma H = 0$

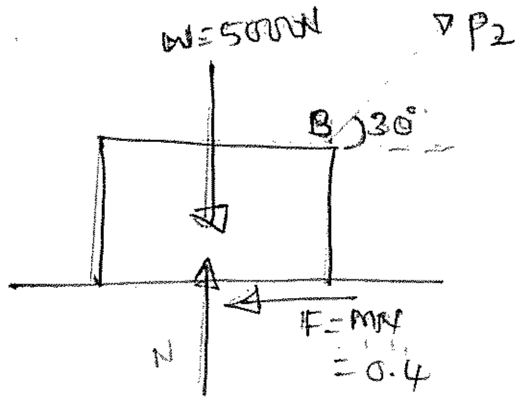
$\cancel{30} P_1 \cos 30^\circ - 0.4N = 0 \rightarrow (2)$

solving (1) and (2)

$\boxed{P_1 = 3002.88 \text{ N}}$

$\boxed{N = 6501.4 \text{ N}}$

(ii)



$\Sigma V = 0$

$+P_2 \sin 30^\circ - 5000 + N = 0 \rightarrow (1)$

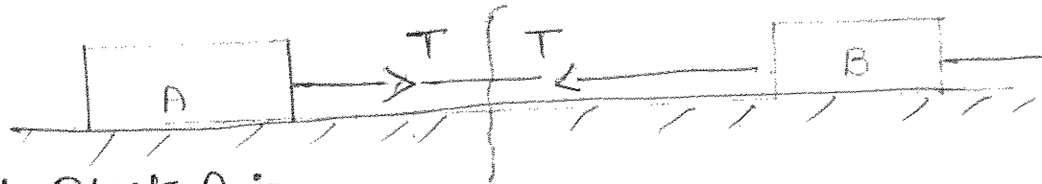
$\Sigma H = 0$

$P_2 \cos 30^\circ - 0.4N = 0 \rightarrow (2)$

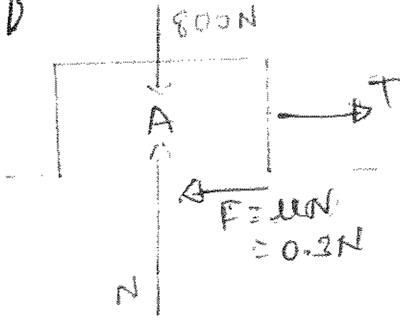
$\boxed{P_2 = 1876.12 \text{ N}}$ $\boxed{N = 4061.93}$

In the (ii) case less effort is required.

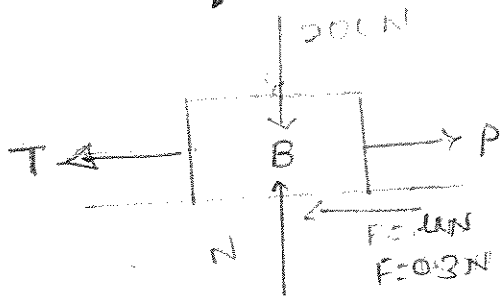
2) The blocks A and B weighs 800N and 200N respectively are connected by a cable. Find the force required to make the blocks move towards right. Find also the tension in the cable. The Co-eff. of friction b/w the blocks and the plane is 0.3.



FBD of Block A :-



FBD of Block B :-



$\Sigma V = 0$

$-800 + N = 0$

$\boxed{N = 800}$

$\Sigma H = 0$

$T - 0.3N = 0$

$T = 0.3 \times 800$

$\boxed{T = 240}$

$\Sigma V = 0$

$-200 + N = 0$

$\boxed{N = 200}$

$\Sigma H = 0$

$-T + P - 0.3N = 0$

$-T + P = 0.3 \times 200$

$P = 60 + T$

$P = 60 + 240$

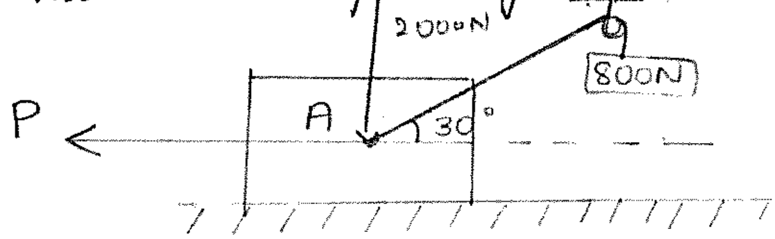
$\boxed{P = 300}$

quired also 5/10

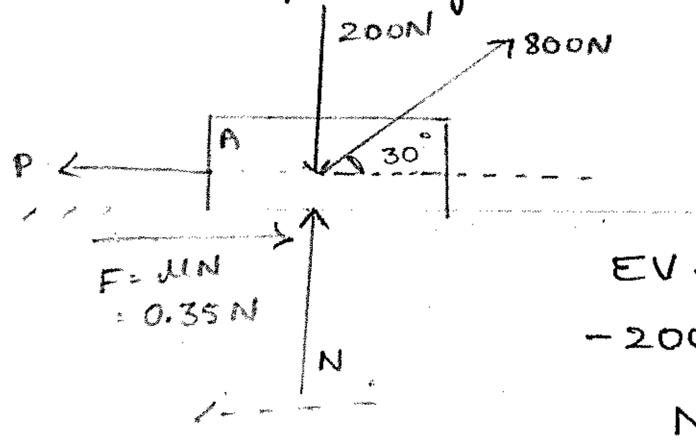
03] The block A shown in the figure weighs 2000N. The chord attached to the block A passes over frictionless pulley and supported a weight of 800N. The coeff of friction b/w the block and the horizontal plane is 0.35. Determine the horizontal force, P if

(i) motion is impending towards left and

(ii) motion is impending towards right.



(i) Motion is impending towards left.



$\Sigma V = 0$

$-2000 + N + 800 \sin 30 = 0$

$N = 2000 - 800 \sin 30$

$\boxed{N = 1600\ Nt}$

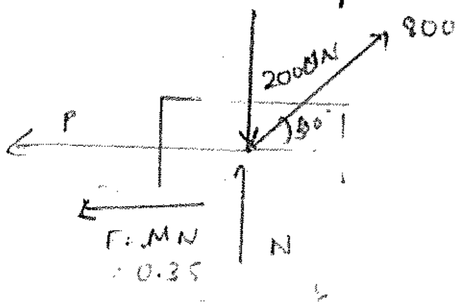
$\Sigma H = 0$

$-P + 0.35N + 800 \cos 30^\circ = 0$

$P = 0.35N + 800 \cos 30^\circ$

$P = 1252.82\ N$

(ii) Motion is impending towards right.



$\Sigma V = 0$

$-2000 + N + 800 \sin 30 = 0$

$N = 1600\ N$

$\Sigma H = 0$

$-P - 0.35N + 800 \cos 30^\circ = 0$

$P = 0.35 \times 1600 - 800 \cos 30^\circ$

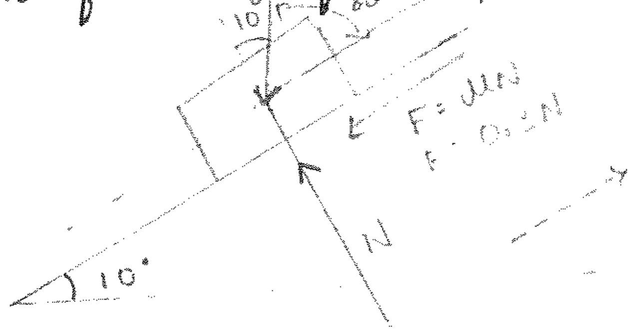
$P = + 132.82\ N$

4] A block weighing 10KN rests on a plane inclined at 10° to the horizontal. If the co-eff of friction is 0.3 find the force required to push the block up the plane, when the line of action of the force is

(i) Parallel to the plane

(ii) Horizontal.

(i) When the line of action of force is parallel to plane



$\Sigma F_{\perp r} = 0$

$-10\sin 80 + N = 0$

$\boxed{N = 9.85\ N}$

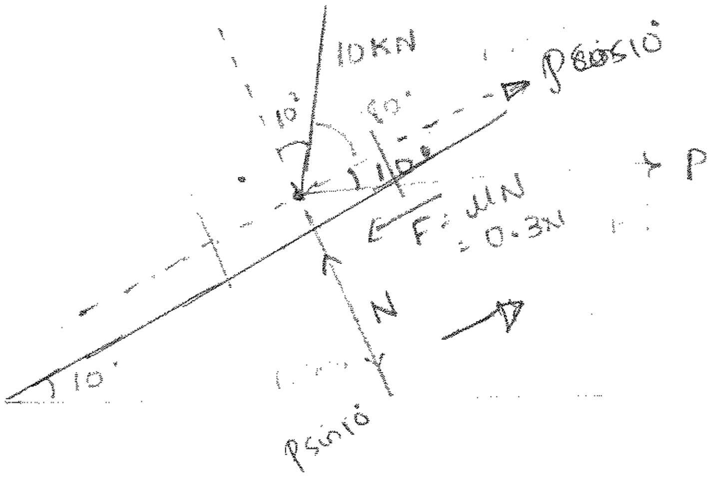
$\Sigma F_{\parallel} = 0$

$P - 10\cos 80 - 0.3N = 0$

$P = 10\cos 80 + 0.3(9.85)$

$P = \underline{4.69\ KN}$

(ii) Horizontal.



$\Sigma F_{\perp r} = 0$

$\cancel{+P\cos 10}$ $\quad -P\sin 10 + N - 10\sin 80° = 0$

$-P\,\cancel{\sin} - 10\sin 80 + N = 0$

$-P\sin 10 + N = 10\sin 80 \rightarrow (1)$

$\Sigma F_{\parallel} = 0$

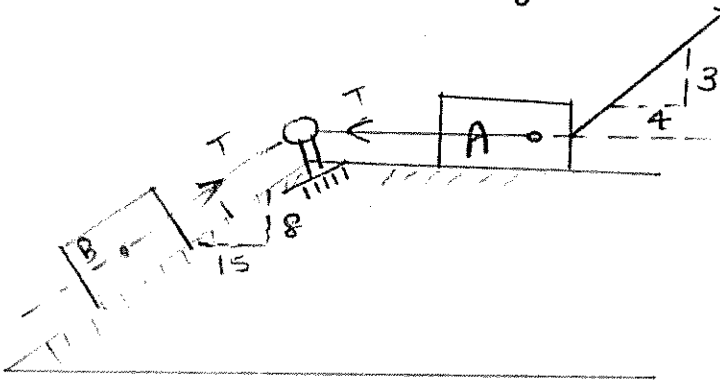
$P\cos 10 - 0.3N - 10\cos 80° = 0$

$P\cos 10 - 0.3N = 10\cos 80.$

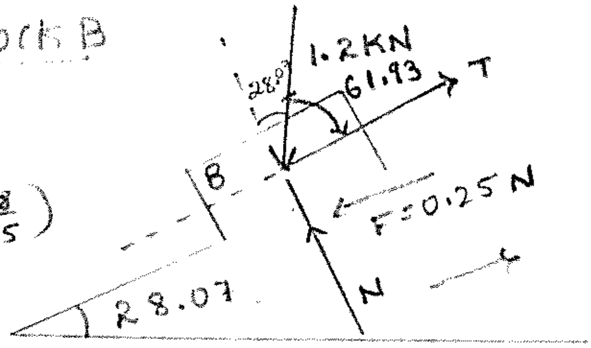
$P = \underline{\underline{5.02}}$

$N = \underline{\underline{10.72}}$

5] Find the value of force P of to be applied to the system of blocks to have impending motion to the ri[ght]. The coefficient of friction for surfaces of contact of block A and B are respectively 0.2 and 0.25. The blocks A weighs 3KN and the block B weighs 1.2 KN.



FBD of Block B



$\theta = \tan^{-1}\left(\frac{8}{15}\right)$

$\theta = \underline{28.07}$

$\Sigma F_{\perp r} = 0$

$-1.2 \sin 61.93 + N = 0$

$N = 1.2 \sin 61.93$

$\boxed{N = 1.05\ N}$

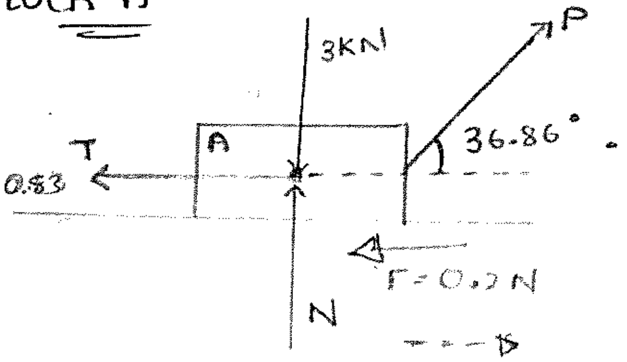
$\Sigma F_{\parallel} = 0$

$T - 1.2 \cos 61.93 - 0.25 N = 0$

$T = 1.2 \cos 61.93 + 0.25 \times 1.$

$T = \underline{0.82\ KN}$

FBD of Block A



$\Sigma V = 0$

$-3 + N + P\sin 36.86° = 0$

$\cancel{N = -P}$

$P \sin 36.86 + N = 3$

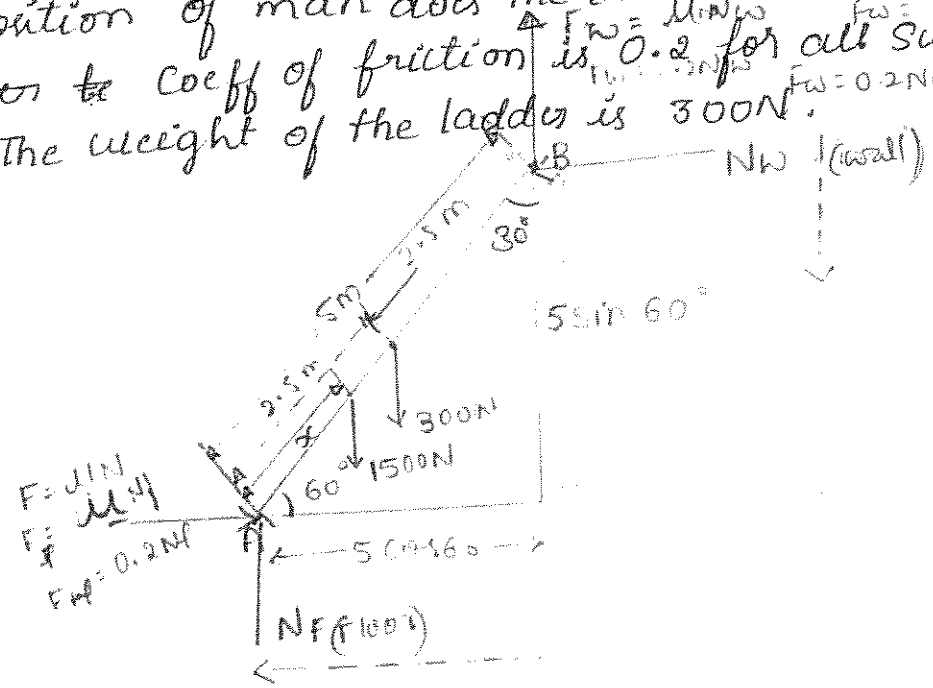
$\Sigma H = 0$

$-0.2N + P\cos 36.86 - 0.83 =$

$+P\cos 36.86 - 0.2N = 0.83$

$\boxed{P = 1.55}$ $\boxed{N = 2.06}$

6) A ladder 5m long is supported by a smooth wall and is resting on a horizontal floor. It is inclined at 30° to the vertical. A man of weight 500N wishes to carry a load of 1000W up the ladder. At what position of man does the ladder tends to sleep? The ~~ladder to~~ coeff of friction is 0.2 for all surface of contact. The weight of the ladder is 300N.



$\Sigma H = 0$

$-N_w + 0.2 N_f = 0 \rightarrow (1)$

$\Sigma V = 0$

$0.2 N_w + N_f - 1500 - 300 = 0$

$0.2 N_w + N_f = 1800 \rightarrow (2)$

By solving (1) and (2)

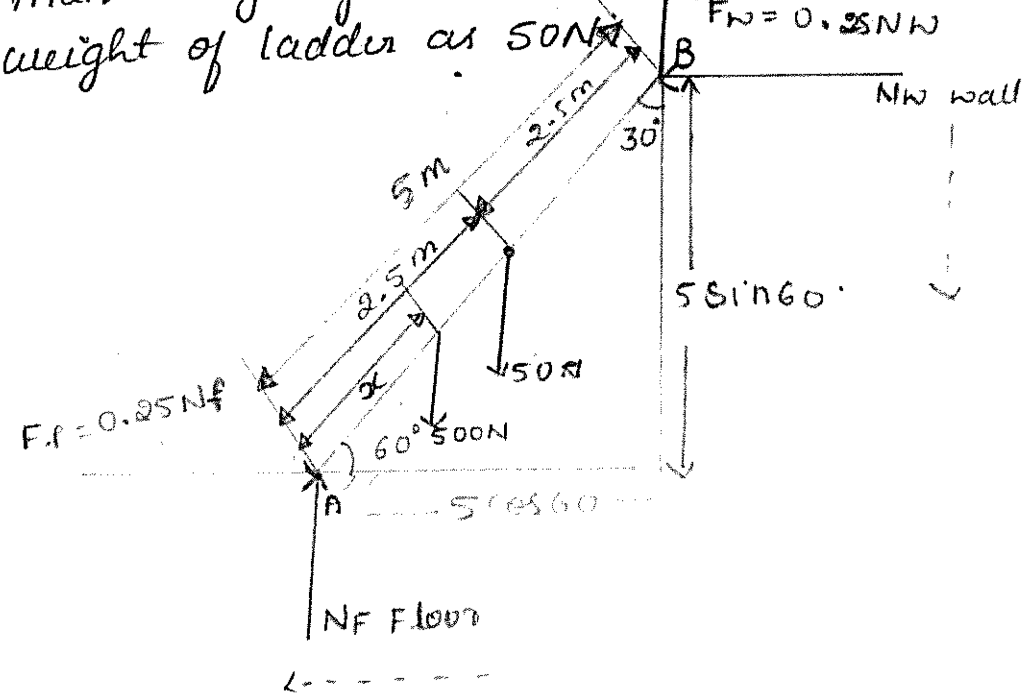
$N_w = \underline{346.15N}$ $\quad N_f = \underline{1730.76N}$

$\Sigma M_A = 0$

$-(346.15 \times 5 \sin 60) - (0.2 \times 346.15 \times 5 \cos 60) + (300 \times 2.5 \cos 60)$
$+ (1800 \times x \cos 60) = 0$

$\boxed{x = 1.73 \text{ m from } A}$

7] A ladder of length 5m rests against a vertical wall making an angle of 60° with the horizontal between ladder is 0.25. A man weighing 500N asunds the ladder slips? Take the weight of ladder as 50N.



$\Sigma H = 0$

$$-N_W + 0.25 N_f = 0 \rightarrow (1)$$

$\Sigma V = 0$

$$0.25 N_W + N_f - 500 - 50 = 0$$

$$0.25 N_W + N_f = 550 \rightarrow (2)$$

By solving (1) and (2)

$N_W = 129.41\,N$ $\quad N_f = 517.64\,N$

$\Sigma M_A = 0$

$$-(129.41 \times 5\cos 60) - (0.2 \times 129.41 \times 5\cos 60) + (50 \times 2.5\cos 60) + (500 \times x\cos 60) = 0.$$

$$-323.53 + 500 x \cos 60 = 0$$

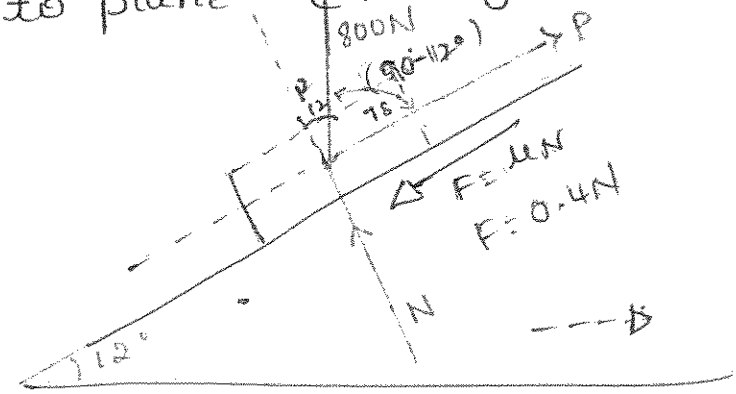
$$500 x \cos 60 = 323.53$$

500

5 cos 60)

4]a] A block weighing 800N rests on a plane inclined at 12° to the horizontal. If the co-efficient of friction is 0.4. find the forces required to push the block of the plane. When the line of action of force is
(i) parallel to plane (ii) Horizontal.

(i)



$\Sigma F_{\perp r} = 0$

$\cancel{12\sin} - 800\sin 78 + N = 0$

$N = 800\sin 78°$

$\boxed{N = 782.5\ N}$

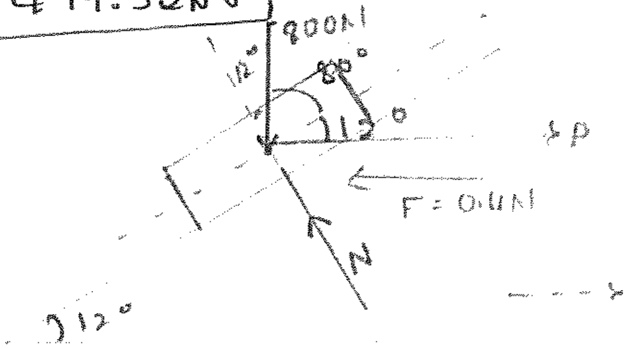
$\Sigma F_{\parallel} = 0$

$-800\cos 78 + P - 0.4N = 0$

$P = 0.4(782.5) + 800\cos 78$

$\boxed{P = 479.32kN}$

(ii)



$\Sigma F_{\perp r} = 0$

$-P\sin 12 + N - 800\sin 80° = 0$

$-P\sin 12 + N = 800\sin 78$

$\Sigma F_{\parallel} = 0$

$P\cos 12 - 0.4N \mp 800\cos 80° = 0$

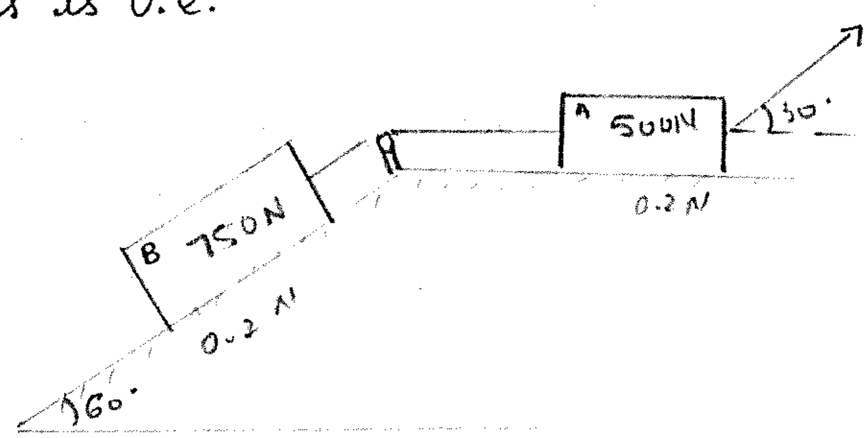
$P\cos 12 - 0.4N = +800\cos 78°$

~~P = 507.33~~

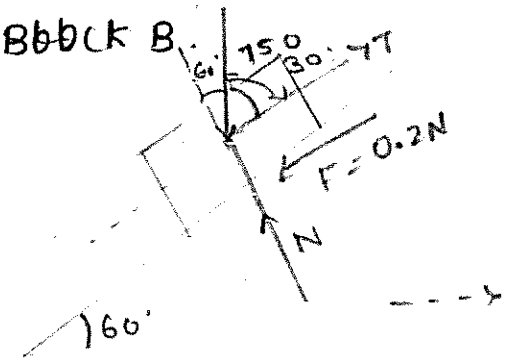
~~N = 893.5~~

P = 535.58N
N = 893.87N

5 a] Determine the value of force P in the system shown in the figure to cause the motion impend to the right. Assume the pully is smothed and coeff of friction b/w the other surfaces is 0.2.



FBD of Block B



$\Sigma F_{\perp r} = 0$

$-750 \sin 30 + N = 0$

$\boxed{N = 375N}$

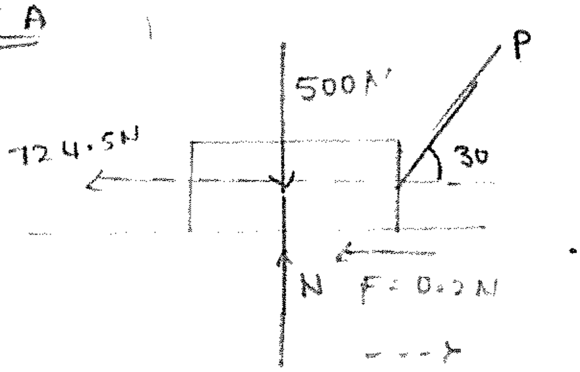
$\Sigma F_{\parallel} = 0$

$T - 750 \cos 30 - 0.2N = 0$

$T = 750 \cos 30^\circ + 0.2 \times 375$

$\boxed{T = 724.5\ N}$

Block A



$\Sigma V = 0$

$-500 + P \sin 30 + N = 0$

$+P \sin 30 + N = 500$

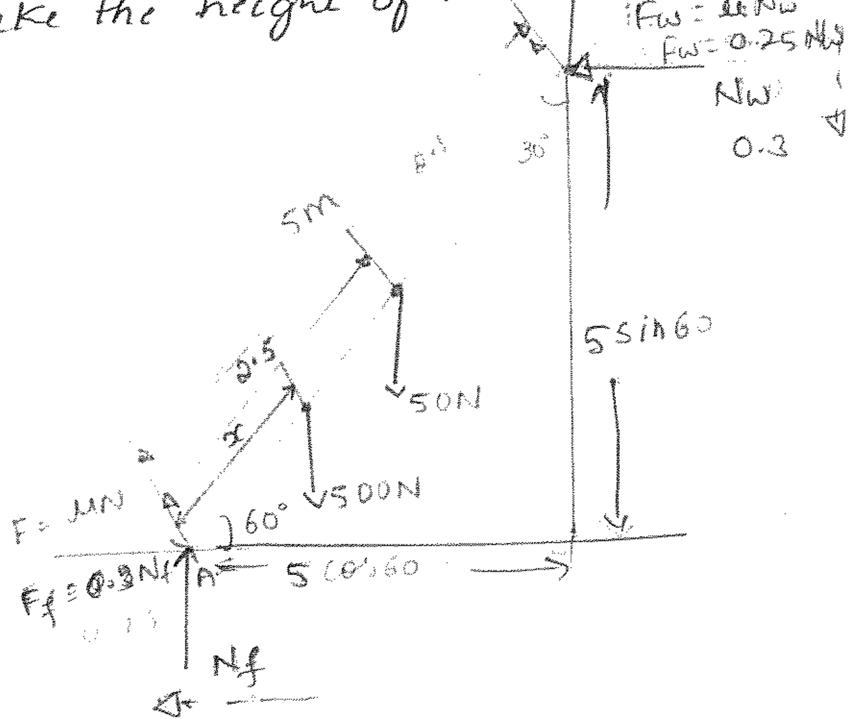
$\Sigma H = 0$

$-0.2N + P \cos 30^\circ - 724.5 = 0$

$+P \cos 30^\circ - 0.2N = 724.5$

$\boxed{P = 853.4}$ $\boxed{N = 73.25}$

7] A ladder of length 5m rests against a wall making an angle of 60° with the horizontal. The co-eff of friction between wall and ladder is 0.3 and floor and ladder is 0.25. A man weighing 500N ascends the ladder. How high will he be able to go before the ladder slips? Take the height of the ladder is 50N.



ΣH = 0

$-N_W + 0.25 N_f = 0 \rightarrow (1)$

ΣV = 0

$0.3 N_W + N_f = 550 \rightarrow (2)$

By solving (1) and (2)

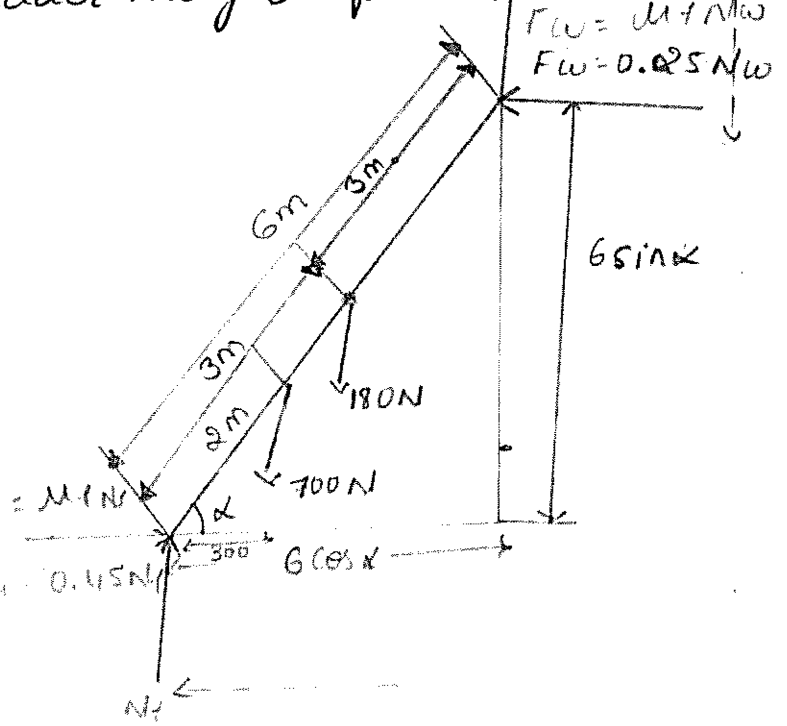
$\boxed{N_W = 127.90N}$ $\boxed{N_f = 511.62 N}$

ΣMA = 0

$-(127.90 \times 5 \sin 60) - (0.3 \times 127.90 \times 5 \cos 60) + (50 \times 2.5 \cos 60) + (500 \times x \cos 60) = 0$

$\boxed{x = 2.349 M}$

8] A ladder of length 6m rests against a vertical wall. The ladder weighs 180N. The coeff of friction b/w the wall and ladder is 0.25 and floor and ladder is 0.45. If a person weighing 700N stands at a distance of 2m from the bottom of the ladder, determine the least value of the angle w.r.t horizontal at which the ladder may be placed without slipping.



$\Sigma H = 0$

$-N_w + 0.45 N_f = 0$ → ①

$\Sigma V = 0$

$0.25 N_w + N_f - 180 - 700 = 0$

$0.25 N_w + N_f = 880$ → ②

By solving ① & ②

$N_w = \underline{355.95\ N}$ $\quad N_f = \underline{791.01\ N}$

$\Sigma M_A = 0$

$-(355.95 \times 6 \sin\alpha) - (0.25 \times 355.95 \times 6 \cos\alpha) + (180 \times 3 \cos\alpha) + (700 \times 2 \cos\alpha) = 0$

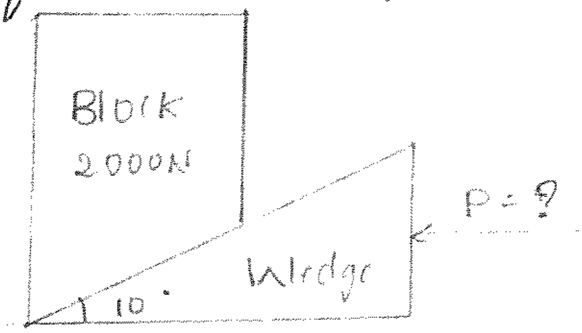
$1406.07 \cos\alpha = 2135.7 \sin\alpha$

$\sin\alpha = 1406.07$

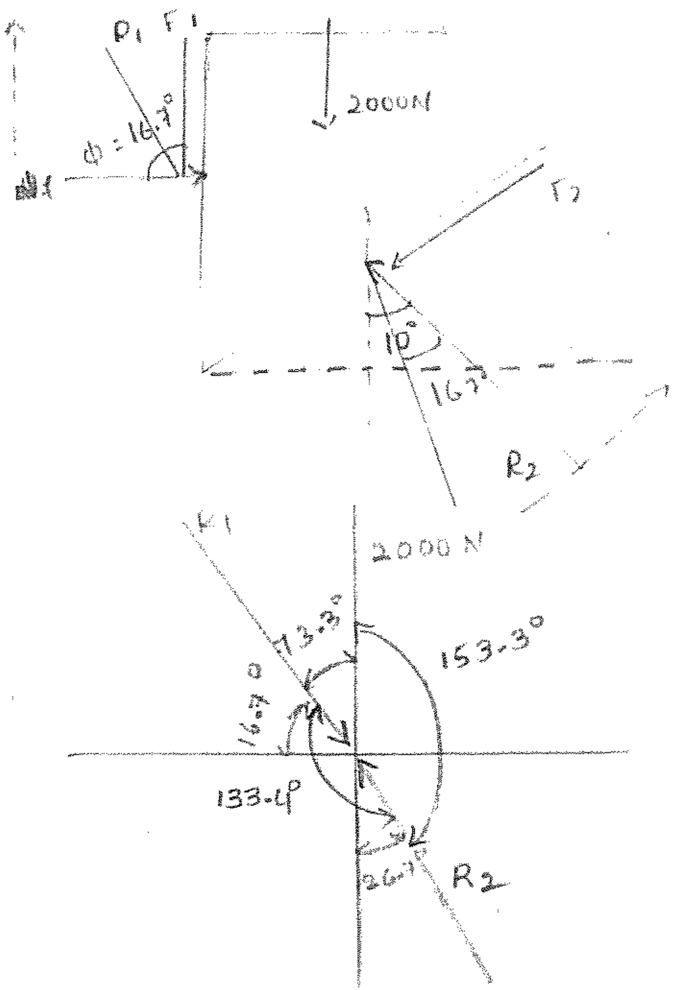
$\tan\alpha = 0.658$

$\boxed{\alpha = 33.36^\circ}$

9] A block is place over a 10° wedge and so leaning against a vertical wall as shown in the figure. The weight of the block is 2000N. Find the minimum force P required to raise the block. The co-efficient of friction is 0.3 for all the contact of force.



FBD of Block :-



$\mu = 0.3$

But $= \mu = \tan\phi$

$\phi = \tan^{-1}(0.3)$

$\boxed{\phi = 16.7^\circ}$

using Lami's theorem

$$\frac{R_1}{\sin 153.3} = \frac{2000}{\sin 133.4} = \frac{R_2}{\sin 73.3}$$

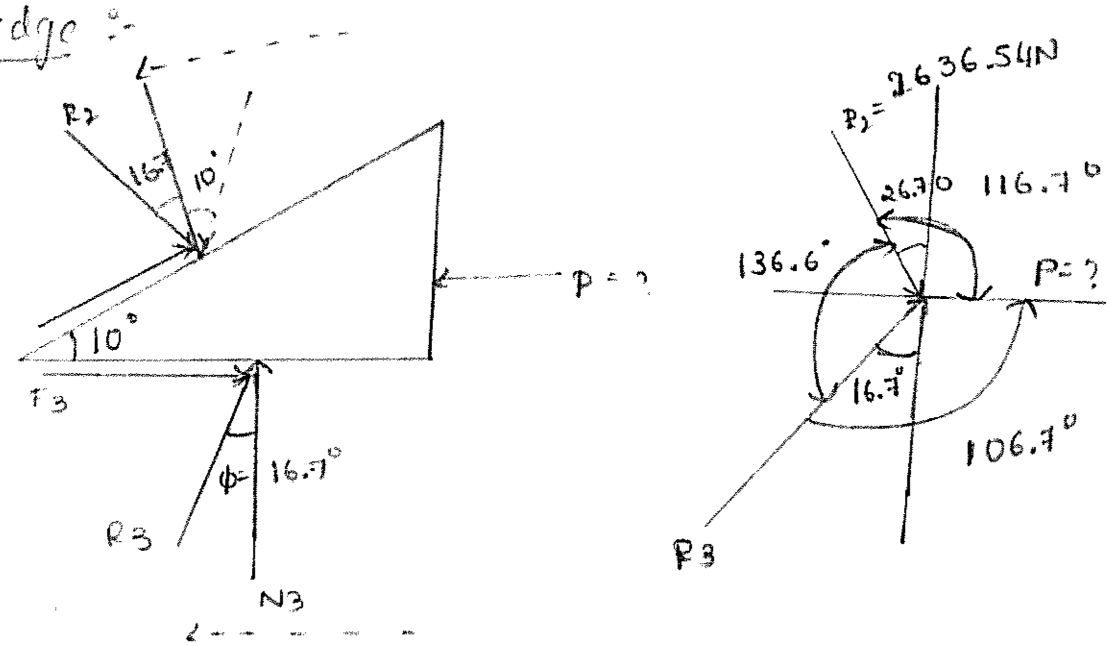
$$R_1 = \left(\frac{2000}{\sin 133.4}\right) \times 153.34$$

$$\boxed{R_1 = 1236.81N}$$

$$R_2 = \left(\frac{2000}{\sin 133.4}\right) \times \sin 73.3$$

$$\boxed{R_2 = 2636.54N}$$

FBD of Wedge :-



By using Lami's theorem

$$\frac{P}{\sin 136^\circ} = \frac{2636.54}{\sin 106.7} = \frac{R_3}{\sin 116.7}$$

$$\frac{P}{\sin 136^\circ} = \frac{2636.54}{\sin 106.7} = 1$$

$$P = \frac{2636.54}{\sin 106.7} \times \sin 136^\circ$$

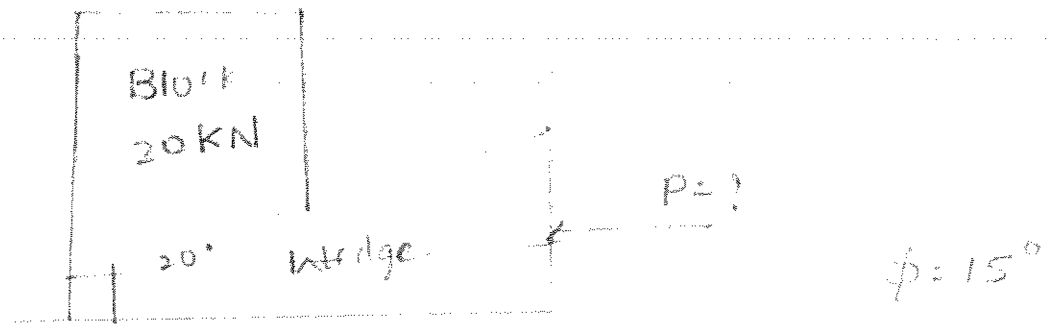
$$\boxed{P = 1891.3 N}$$

$$R_3 = \frac{2636.54}{\sin 106.7} \times \sin 116.7.$$

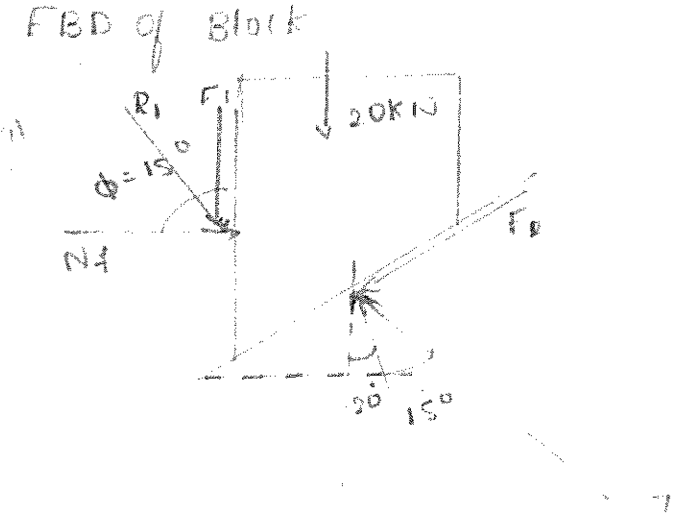
$$\boxed{R_3 = 2459.13 N}$$

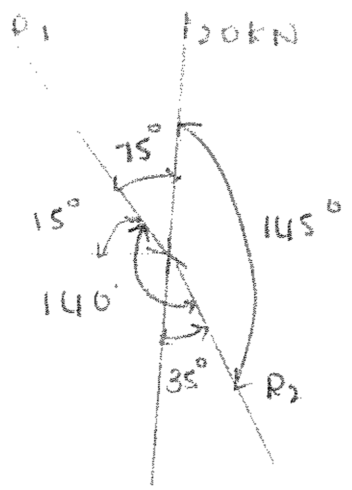
10] Determine the force 'P' required to start the movement of the wedge as shown in the figure. The angle of friction for all surfaces of contact is 15°.

$\phi = 15^\circ$



FBD of Block





By using lami's theorem.

$$\frac{R_1}{\sin 145°} = \frac{20}{\sin 140} = \frac{R_2}{\sin 75°}$$

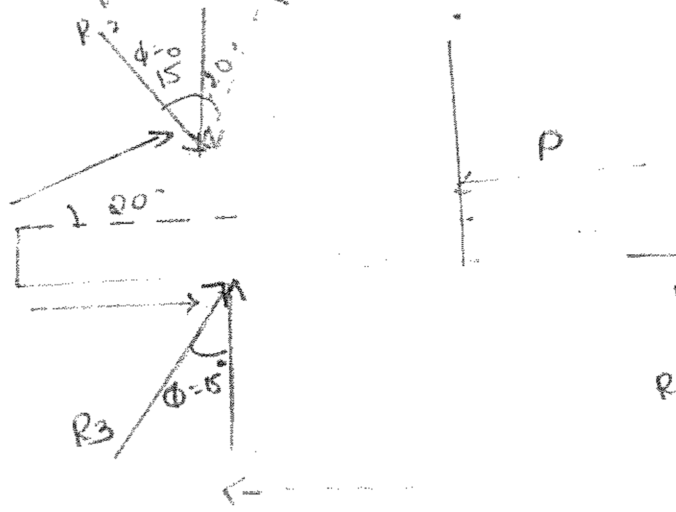
$$R_1 = \frac{20}{\sin 140} \times \sin 145$$

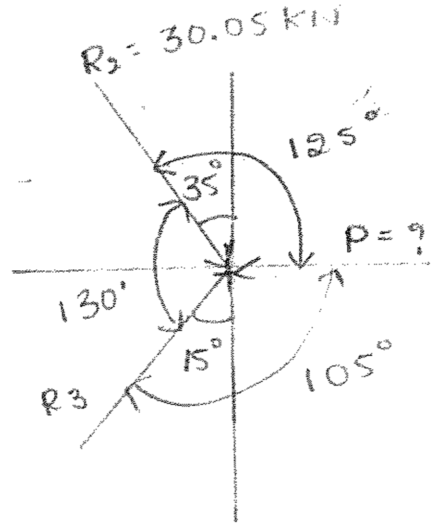
$$\boxed{R_1 = 17.84\ KN}$$

$$R_2 = \frac{20}{\sin 140} \times \sin 75°$$

$$\boxed{R_2 = 30.05\ KN}$$

FBD of wedge





By using lamis theorem

$$\frac{P}{\sin 130^\circ} = \frac{30.05}{\sin 105^\circ} = \frac{R_3}{\sin 125^\circ}$$

$$\frac{P}{\sin 130} = \frac{30.05}{\sin 105}$$

$$P = \frac{30.05}{\sin 105} \times \sin 130$$

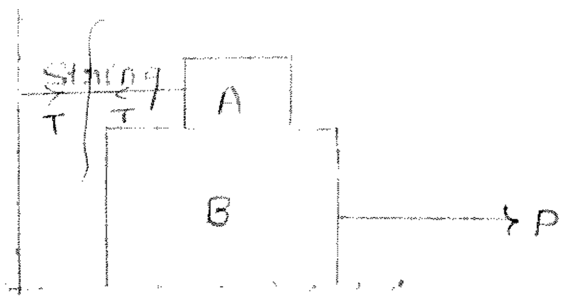
$$\boxed{P = 23.83 KN}$$

$$\frac{R_3}{\sin 125} = \frac{30.05}{\sin 105^\circ}$$

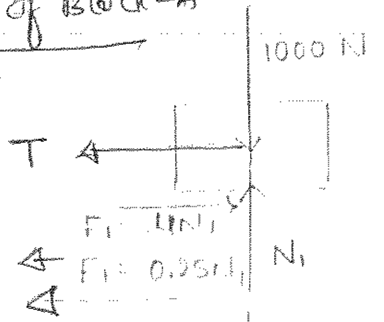
$$R_3 = \frac{30.05}{\sin 105} \times \sin 125$$

$$\boxed{R_3 = 25.48 KN}$$

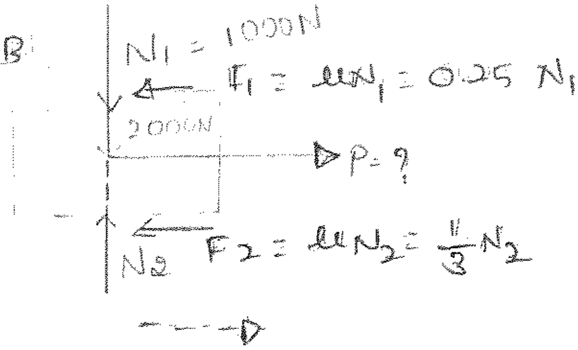
11] Block A weighing 1000N rests over block B which weighs 2000 N as shown in the figure. Block A is tied to wall with a horizontal string. If the Co-efficient of friction between blocks A and B is 0.25 and between B and the floor is $\frac{1}{3}$. What should be the value of P to move the block B if P is horizontal.



FBD of Block-A



FBD of Block-B



From FBD of Block A :-

$\Sigma V = 0$

$N_1 - 1000 = 0$

$\boxed{N_1 = 1000\,N}$

$\Sigma H = 0$

$0.25 N_1 - T = 0$

$T = 0.25 \times 1000$

$\boxed{T = 250\,N}$

From FBD of Block B :-

$\Sigma V = 0$

$N_2 - 1000 - 2000 = 0$

$\boxed{N_2 = 3000\,N}$

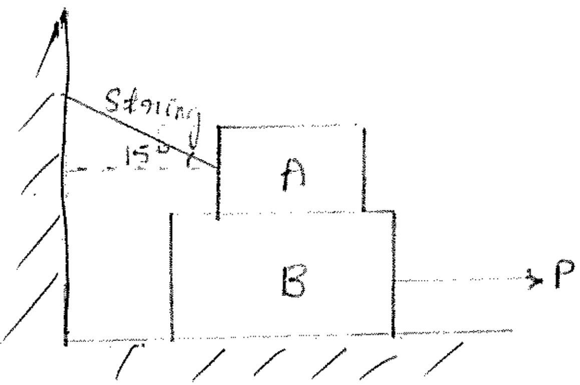
$\Sigma H = 0$

$P - 0.25 N_1 - \frac{1}{3} N_2 = 0$

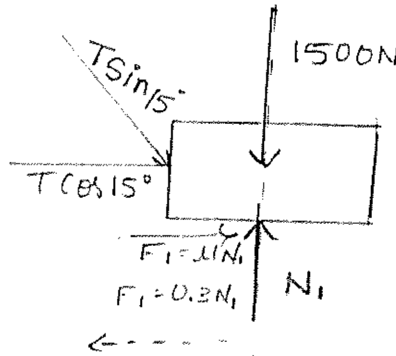
$P = 0.25 \times 1000 + \frac{1}{\cancel{3}} \times \overset{1000}{\cancel{3000}}$

$\boxed{P = 1250\,N}$

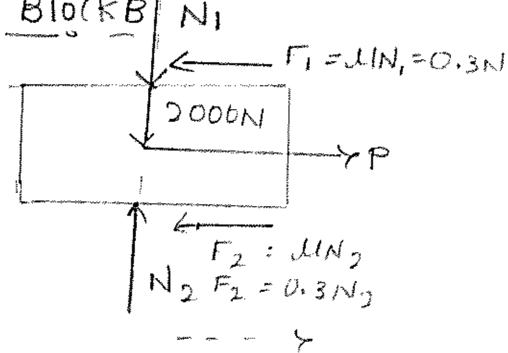
12] Determine the tension in the string and the value of P, when the block B impends moving to the right as shown in the figure. Take the co-efficient of friction for all contact surfaces as 0.3. The block A weighs 1500N and the block B weighs 2000N.



FBD of Block A



FBD of Block B



From FBD of Block A

$\Sigma V = 0$

~~$N_1 - 1500$~~

$T\sin 15 + N_1 - 1500 = 0$

$N_1 + T\sin 15 = 1500$

$\Sigma H = 0$

$-T\cos 15 + 0.3 N_1 = 0$

$0.3 N_1 + T\cos 15 = 0$

$\boxed{N_1 = 1388.39N}$ $\boxed{T = 431.21N}$

From FBD of Block B :-

$\Sigma V = 0$

$N_2 - 1388.39 - 2000 = 0$

$\boxed{N_2 = 3388.3}N$

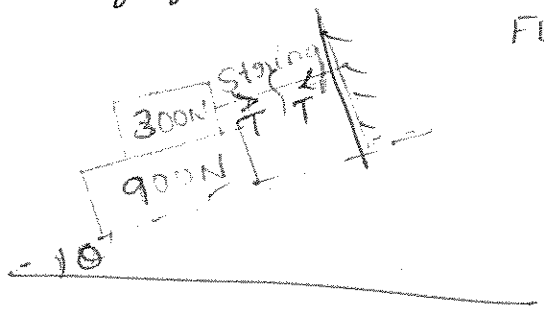
$\Sigma H = 0$

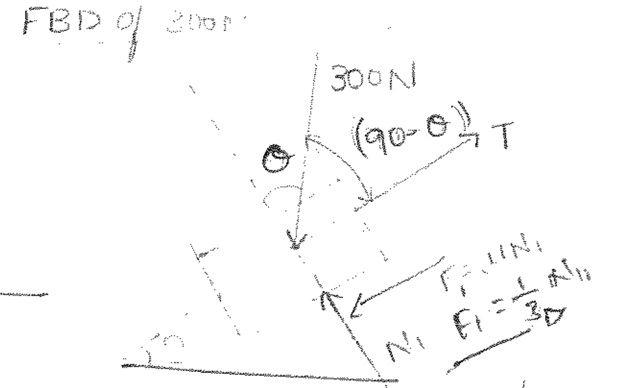
$P - 0.3N_1 - 0.3N_2 = 0$

$P = 0.3 \times 1388.39 + 0.3 \times 3388.3$

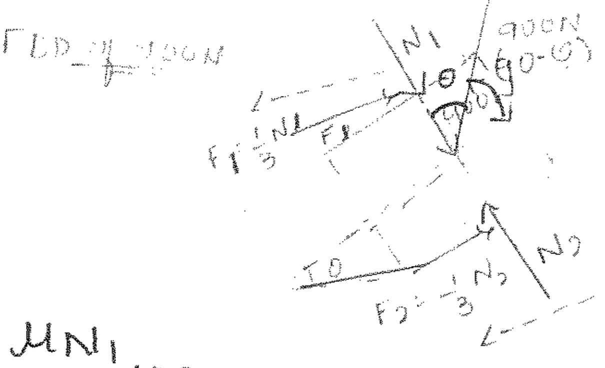
$\boxed{P = 1433.98}$

13] What should be the value of θ, that will make the motion of 900N block down the plane to impend ? The co-efficient of friction for all contact surface is $\frac{1}{3}$.





From FBD of 300N



$\Sigma F_{\perp r} = 0$

$N_1 - 300\sin(90-\theta) = 0$

$\boxed{N_1 = 300\cos\theta}$

$F_1 = \mu N_1$

$F_1 = \frac{1}{\cancel{3}} \times \overset{100}{\cancel{300}}\cos\theta$

$\boxed{F_1 = 100\cos\theta}$

$\Sigma F_{\parallel k} = 0$

$(= \frac{1}{3} N_1)$

$T - 100\cos\theta - 300\cos(90-\theta) = 0$

$T = 100\cos\theta + 300\sin\theta$

FBD of 900N :-

$\Sigma F_{\perp r} = 0$

$N_2 - N_1 - 900\sin(90-\theta) = 0$

$N_2 = 900\cos\theta + 300\cos\theta$

$N_2 = 1200\cos\theta$

$F_2 = \mu N_2$

$F_2 = \frac{1}{\cancel{3}} \times \overset{400}{\cancel{1200}}\cos\theta$

$\boxed{F_2 = 400\cos\theta}$

$\Sigma F_{\parallel k} = 0$

$F_1 + F_2 \cancel{- 400\cos\theta - 900\cos(90-\theta)} = 0$

$F_1 + F_2 - 900\cos(90-\theta) = 0$

$100\cos\theta + 400\cos\theta - 900\sin\theta = 0$

$500\cos\theta - 900\sin\theta = 0$

the
The

$$500 \cos\theta = 900 \sin\theta$$

$$\frac{\sin\theta}{\cos\theta} = \frac{500}{900}$$

$$\tan\theta = 0.5$$

$$\boxed{\theta = 29.05^\circ}$$

<u>To find T:-</u>

$$T = 100 \cos(29.05) + 300 \cancel{\cos} \sin(29.05^\circ)$$

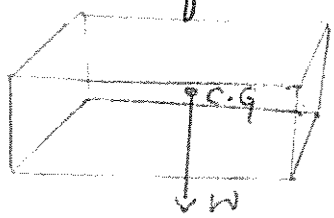
$$\boxed{T = 233.09\,N}$$

UNIT - IV

Centroid of Plane areas.

Centre of gravity (C.G) :-

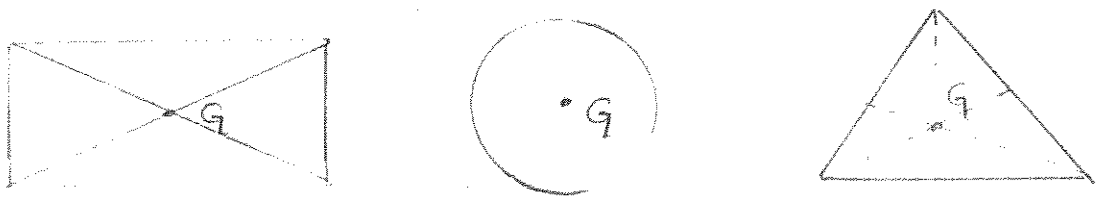
The point through which the whole weight of the body, acts irrespective of the position of the body is known as Centre of gravity.



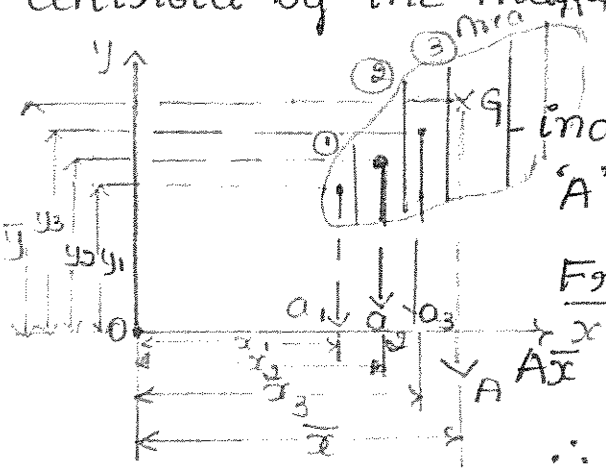
Centroid (G) :-

The point at which the entire area of the plane figure is assumed to be concentrated is called as centroid of the area.

Centroid is a term that is always used for plane figures which has no mass.



Centroid by the method of moment :-



Let $\bar{x}$ and $\bar{y}$ be the co-ordinates of the centroid of the area 'A' from ox and oy axes.

From the principle of moments :

$$A\bar{x} = a_1x_1 + a_2x_2 + a_3x_3 + \cdots$$

$$\therefore \bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3 + \cdots}{A}$$

Where $A = \Sigma a = a_1 + a_2 + a_3 + \cdots$

$$\therefore \boxed{\bar{x} = \frac{\Sigma a x}{\Sigma a}}$$

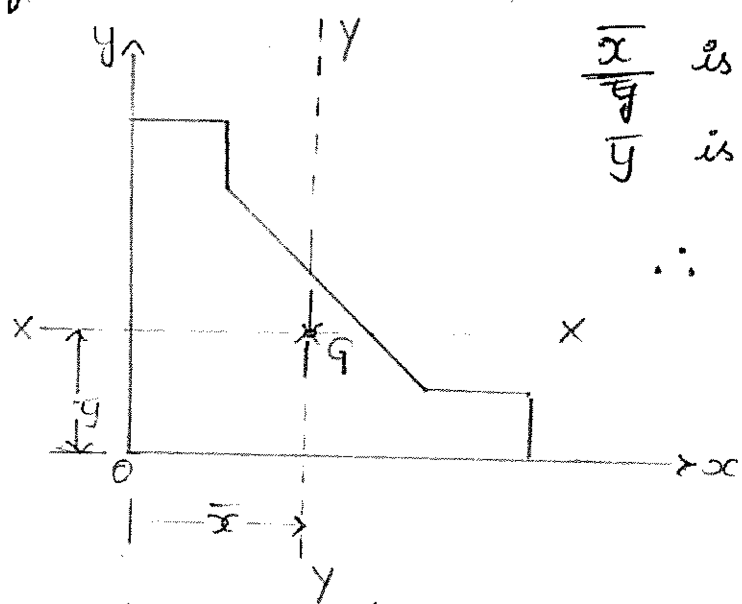
Also, $\boxed{\bar{x} = \dfrac{\int \delta a . x}{\int \delta a}}$ for geometrical figures.

$$\boxed{\bar{y} = \frac{\Sigma a . y}{\Sigma a}}$$

Also $\boxed{\bar{y} = \dfrac{\int \delta a . y}{\int \delta a}}$ for geometrical figures

Axis of Reference :-

The centroid (G) of an area is calculated with reference to some assumed axis of reference.



$\bar{x}$ is a measured from Oy

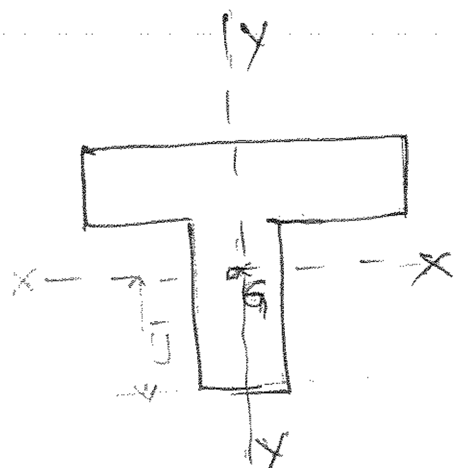
$\bar{y}$ is measured from Ox.

$\therefore$ Ox and Oy are called as axes of reference.

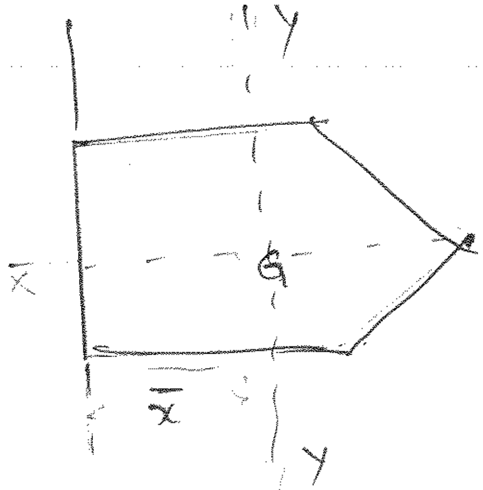
Axis of symmetry :-

It is an axis which divides the area into two equal parts such that each part is a mirror image of the other.

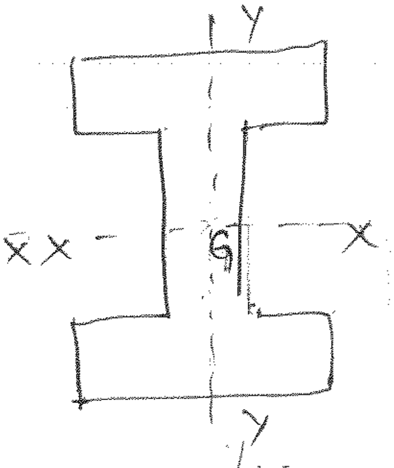
centroid (G) of an area always lies on the axis of symmetry. If there are two axes of symmetry then the point of intersection of these two axes will be the centroid of the plane figure.



T-Section
Symmetrical about YY axis



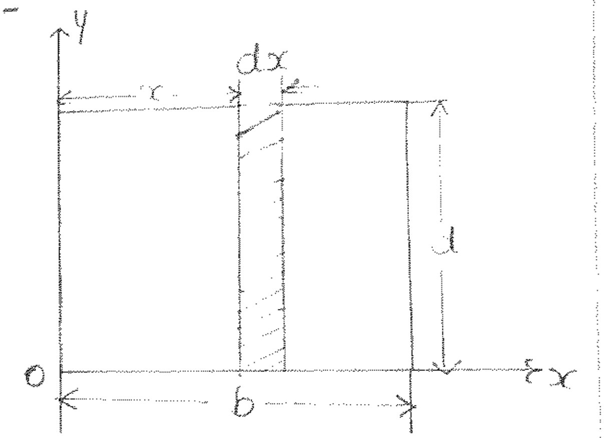
Symmetrical about XX axis



I-Section
Symmetrical about BOTH XX & YY axis

Determination of centroid of simple figures from method of integration (or) from first principles:-

1] Centroid of a rectangle :-



Area of the element,

$\delta a = d \times dx$

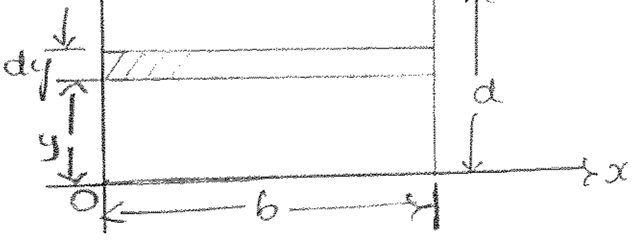
$$\bar{x} = \frac{\int \delta a \cdot x}{\int \delta a}$$

$$\int_0^b \delta a \cdot x = \int_0^b (d \times dx)x = d\int_0^b x \cdot dx = d\left[\frac{x^2}{2}\right]_0^b = d\left[\frac{b^2}{2}\right]$$

ax. d $\int \delta a = bd$
(Area of rectangle)

$$\therefore \bar{x} = \frac{d(b^2/2)}{b \cdot d}$$

$$\boxed{\bar{x} = \frac{b}{2}}$$



Area of the element,

$\delta a = b \times dy$

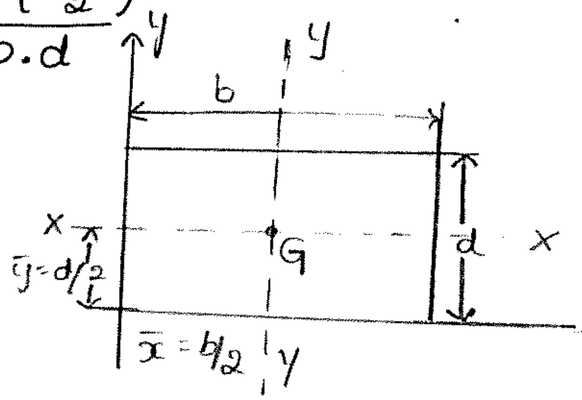
$$\bar{y} = \frac{\int \delta a \cdot y}{\int \delta a}$$

$$\int_0^d \delta a \cdot y = \int_0^d (b \times dy) \cdot y$$

$$= b\int_0^d y \cdot dy = b\left[\frac{y^2}{2}\right]_0^d = b\left(\frac{d^2}{2}\right)$$

Area of the rectangle $= \int \delta a = b \cdot d$

$$\therefore \bar{y} = \frac{b \cdot \left(\frac{d^2}{2}\right)}{b \cdot d}$$



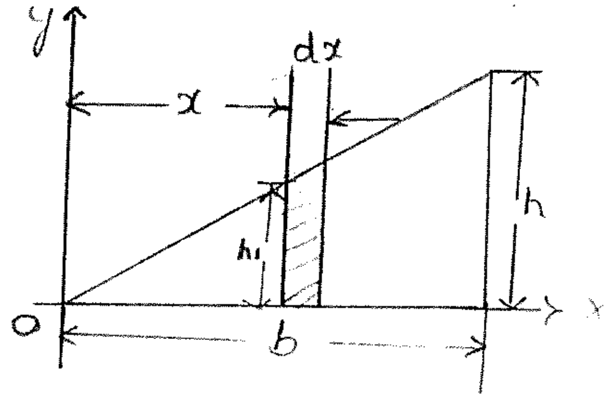
$$\therefore \boxed{\bar{y} = \frac{d}{2}}$$

2] Centroid of a triangle :-

From similar triangles,

$$\frac{h_1}{h} = \frac{x}{b},$$

$$\therefore h_1 = \frac{x}{b} \times h.$$



Area of the element :-

$$\delta a = h_1 \times dx$$
$$= \left(\frac{x}{b} \times h\right) \times dx$$

$$\int_0^b \delta a \cdot x = \int_0^b \left(\frac{x}{b} \times h\right) dx \cdot x = \frac{h}{b}\int_0^b x^2 \cdot dx$$

$$= \frac{h}{b}\left[\frac{x^3}{3}\right]_0^b = \frac{h}{b}\left(\frac{b^3}{3}\right) = \frac{h(b^2)}{3}.$$

Area of triangle $= \int \delta a = \frac{1}{2} \cdot b \cdot h$

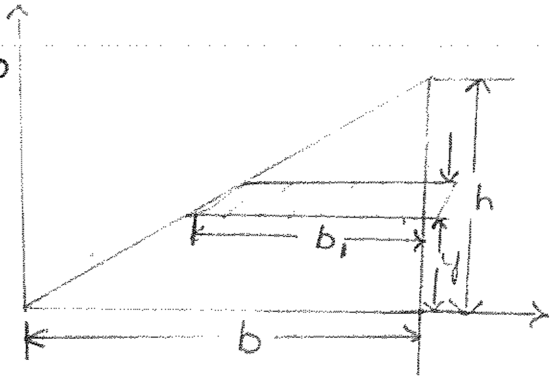
$$\bar{x} = \frac{\int \delta a \cdot x}{\int \delta a} = \frac{\frac{h(b^2)}{3}}{\frac{1}{2} bh}$$

$$\therefore \boxed{\bar{x} = \frac{2}{3} b.}$$ from Oy

From similar triangles,

$$\frac{b_1}{b} = \frac{h-y}{h}$$

$\therefore\ b_1 = \dfrac{(h-y)\,b}{h} = \left(1 - y/h\right) b$



Area of the element,

$$\delta a = b_1 \times dy = \left(1 - \frac{y}{h}\right) b \cdot dy$$

$$\int_0^h \delta a \cdot y = \int_0^h \left(1 - y/h\right) b \cdot dy \cdot y$$

$$= \int_0^h \left(y - \frac{y^2}{h}\right) b \cdot dy = b\left[\frac{y^2}{2} - \frac{y^3}{3h}\right]_0^h$$

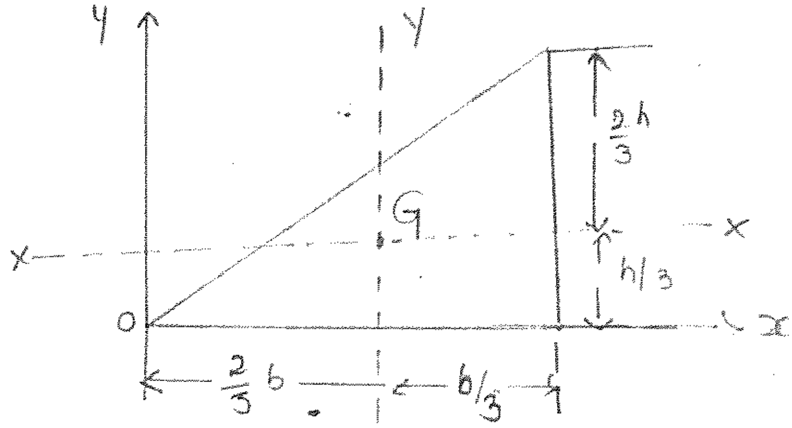
$$= b\left[\frac{h^2}{2} - \frac{h^3}{3h}\right] = b\left[\frac{3h^3 - 2h^3}{6h}\right]$$

$$= b\left(\frac{h^2}{6}\right)$$

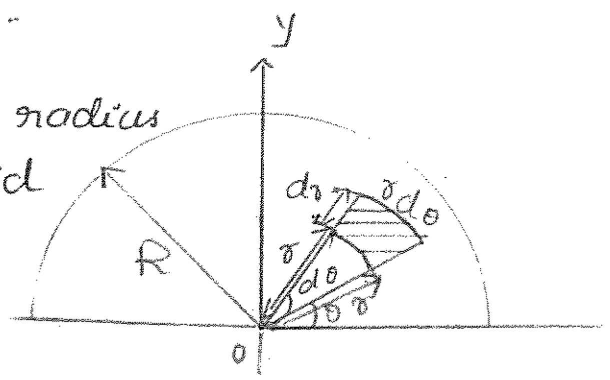
Area of triangle $= \int \delta a = \frac{1}{2} \cdot b \cdot h$

$$\bar{y} = \frac{\int \delta a \cdot y}{\int \delta a} = \frac{\frac{bh^2}{6}}{\frac{1}{2} \cdot bh}$$

$$\therefore \boxed{\bar{y} = \frac{h}{3}} \text{ from the base}$$



3] Centroid of a Semi circle :-



Consider the semicircle of radius R. Due to symmetry centroid must lie on y-axis. Let its distance from diametral axis by $\bar{y}$.

Consider an element at a distance r from the centre of the semicircle, radial width being dr and bound by radii at θ and $(\theta + d\theta)$.

The elemental area may be treated as a rectangle of sides $r\,d\theta$ and dr.

$\therefore$ Area of the element, $\delta a = r.d\theta.dr$

$$\int \delta a . y = \int_0^{\pi}\int_0^{R} (r.d\theta.dr) \times r\sin\theta$$

$$= \int_0^{\pi}\int_0^{R} r^2 . \sin\theta \, dr.d\theta$$

$$= \left[\frac{r^3}{3}\right]_0^R \left[-\cos\theta\right]_0^{\pi} .$$

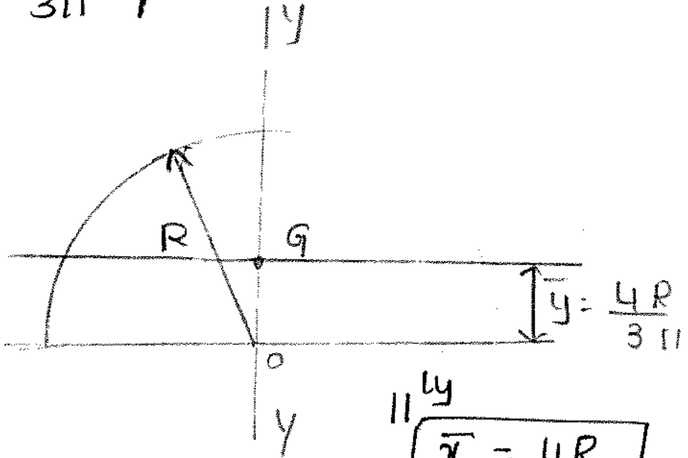
Total moment of area about diametral axis $= \dfrac{R^3}{3}[1+1] = \dfrac{2R^3}{3}$

Area of semicircle :- $\int \delta a = \dfrac{\pi R^2}{2}$

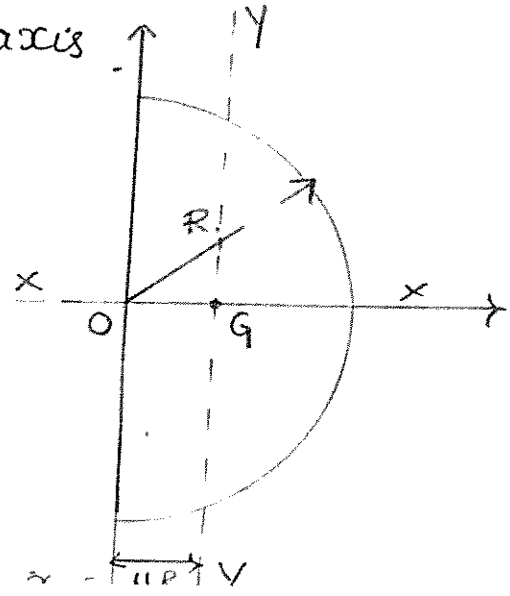
$$\bar{y} = \frac{\int \delta a . y}{\int \delta a} = \frac{\frac{2R^3}{3}}{\frac{\pi R^2}{2}}$$

$$\boxed{\bar{y} = \frac{4R}{3\pi}}$$

$\therefore$ The centroid of the semicircle is at a distance of $\dfrac{4R}{3\pi}$ from the diametral axis.

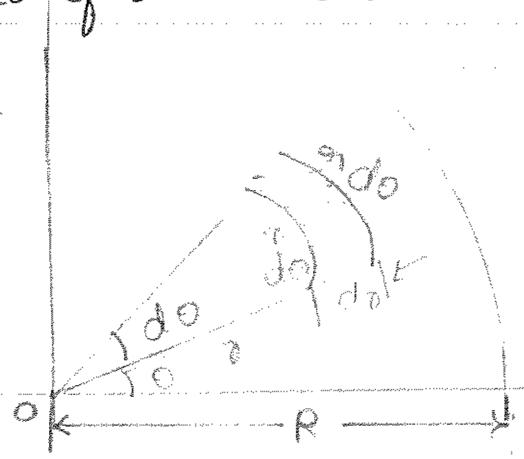


$$\boxed{\bar{x} = \frac{4R}{3\pi}}$$



(4) Centroid of Quadrant of a circle :-

The elemental area may be treated as a rectangle of sides $r\,d\theta$ and dr.



Area of the element,

$$\delta a = r\,d\theta \cdot dr$$

$$\int \delta a \cdot y = \int_0^{\pi/2}\int_0^{R} (r \cdot d\theta \cdot dr) \times r\sin\theta$$

→ Total moment of area about diametral axis.

$$= \int_0^{\pi/2}\int_0^{R} r^2 \cdot dr \cdot \sin\theta \cdot d\theta$$

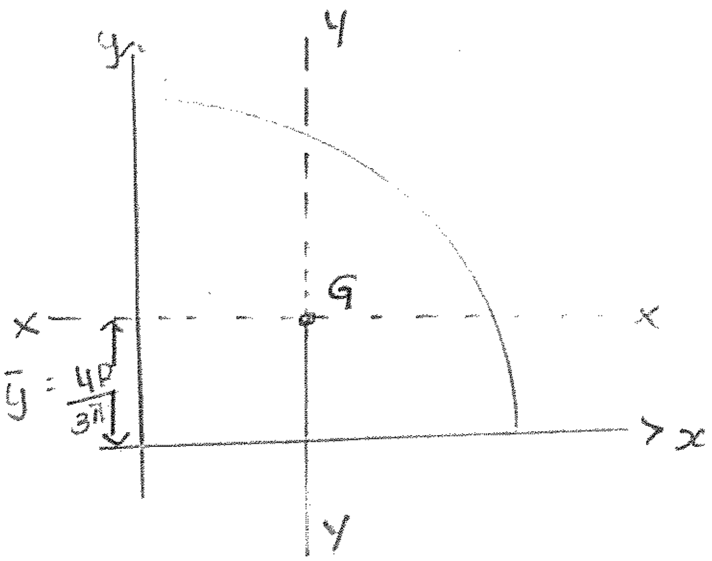
$$= \left[\frac{r^3}{3}\right]_0^R \left[-\cos\theta\right]_0^{\pi/2} = \frac{R^3}{3}\left[0+1\right] = \frac{R^3}{3}$$

Area of the quadrant of a circle $(\int \delta a)$ $= \dfrac{\pi R^2}{4}$

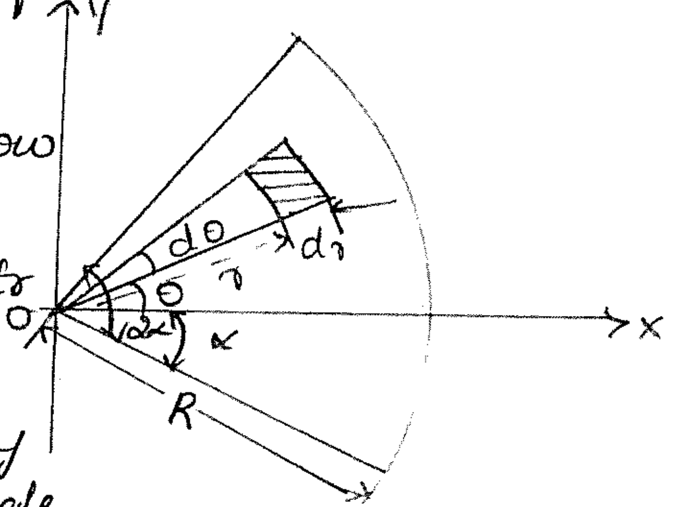
$$\therefore \bar{y} = \frac{\int \delta a \cdot y}{\int \delta a} = \frac{R^3/3}{\pi R^2/4}$$

$$\boxed{\bar{y} = \frac{4R}{3\pi}}$$

The centroid of the quadrant of a circle is at a distance of $\frac{4R}{3\pi}$ from the diametral axis.



(5) Centroid of Sector of a circle :-



Consider the sector of a circle of angle 2α as shown in the figure.

Due to symmetry, centroid lies on x axis.

The elemental area may be treated as a rectangle of sides $r\,d\theta$ and dr.

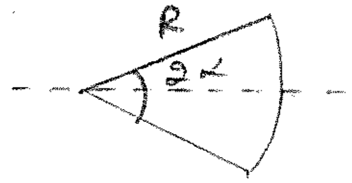
Area of the element, $\delta a = r\,d\theta . dr$

$$\int \delta a . x = \int_{-\alpha}^{\alpha}\int_{0}^{R} (r . d\theta . dr) \times r\cos\theta$$ → Total moment of area about y axis.

$$= \int_{-\alpha}^{\alpha}\int_{0}^{R} r^2 . dr \cos\theta \, d\theta = \left[\frac{r^3}{3}\right]_0^R \left[\sin\theta\right]_{-\alpha}^{\alpha}$$

$$= \frac{R^3}{3} . 2\sin\alpha = \frac{2R^3}{3}\sin\alpha$$

Total area of sector $= \left[\int_{-\alpha}^{\alpha}\int_{0}^{R} r\,dr\,d\theta = \int_{-\alpha}^{\alpha}\left[\frac{r^2}{2}\right]_0^R d\theta\right.$



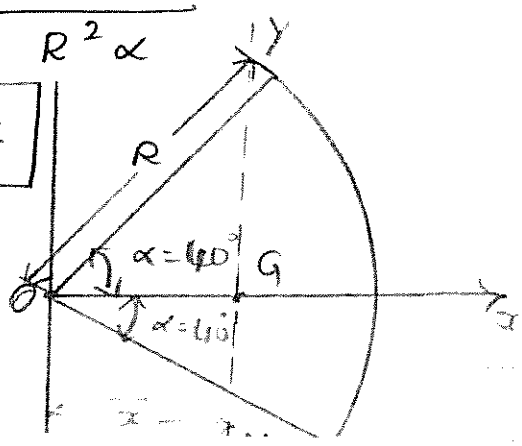
$$\left. = \frac{R^2}{2}\left[\theta\right]_{-\alpha}^{\alpha} = \frac{R^2}{2}\left[2\alpha\right] = R^2 . \alpha\right]$$

Area of Sector $= \frac{1}{2} R^2 (2\alpha)$

$= R^2\alpha$

$$\therefore \bar{x} = \frac{\int \delta a . x}{\int \delta a} = \frac{\frac{2R^3}{3}\sin\alpha}{R^2\alpha}$$

$$\therefore \boxed{\bar{x} = \frac{2R}{3\alpha}\sin\alpha}$$



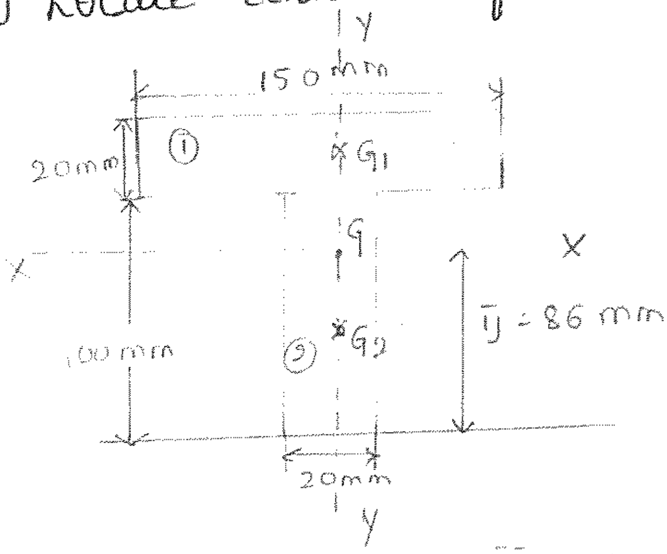
For ex :- $R = 50$ mm , $\alpha = 40°$

$$\bar{x} = \frac{2 \times 50}{3 \times 40 \times \frac{\pi}{180}} \times \sin 40$$

$\bar{x} = 30.60$ mm

Problems :-

1] Locate centroid of T-section shown in the figure.

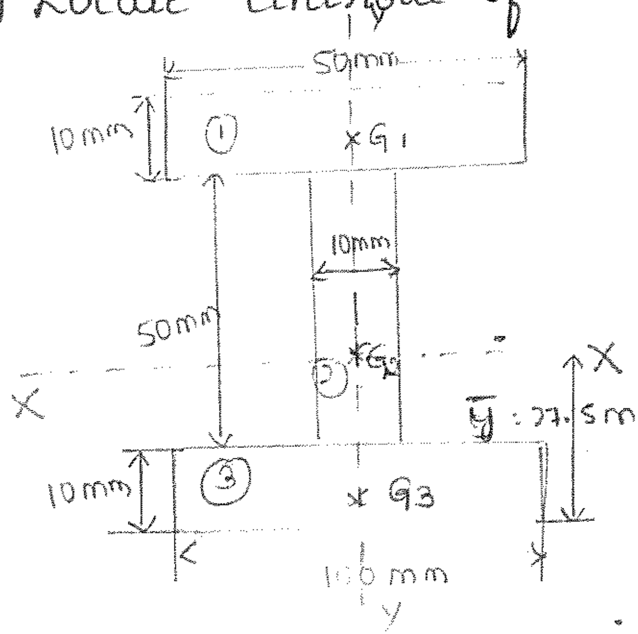


$a_1 = 150 \times 20 = 3000 m^2$	$y_1 = 100 + 10 = 110 mm$
$a_2 = 100 \times 20 = 2000 m^2$	$y_2 = \frac{100}{2} = 50 mm$
$\Sigma a = 3000 + 2000 = 5000 m^2$	$\bar{y} = 86 mm$

$$\bar{y} = \frac{\Sigma a y}{\Sigma a} = \frac{(3000 \times 110) + (2000 \times 50)}{5000}$$

$$\boxed{\bar{y} = 86 mm}$$

2] Locate centroid of the I-section shown in figure.

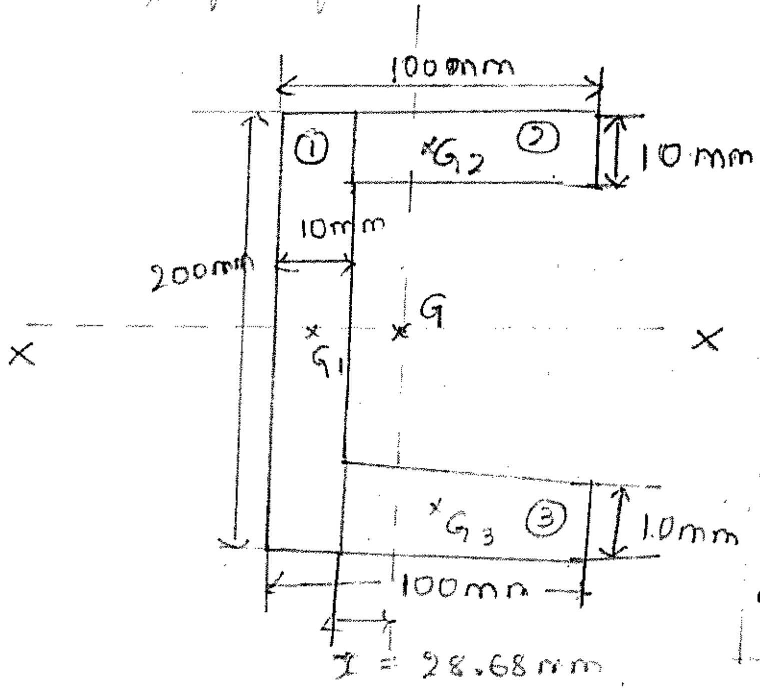


$a_1 = 50 \times 10 = 500 m^2$	$y_1 = 50 + 5 + 10$, $y_1 = 65 mm$
$a_2 = 50 \times 10 = 500 m^2$	$y_2 = \frac{50}{2} + 10 = 35 mm$
$a_3 = 10 \times 100 = 1000 m^2$	$y_3 = 5 mm$
$\Sigma a = 2000 m^2$	$\bar{y} = 27.5$

$$\bar{y} = \frac{\Sigma a y}{\Sigma a} = \frac{(500 \times 65) + (500 \times 35) + (1000 \times 5)}{2000}$$

$$\bar{y} = 27.5 mm$$

③ Locate Centroid of channel section [C-section] shown in the figure.

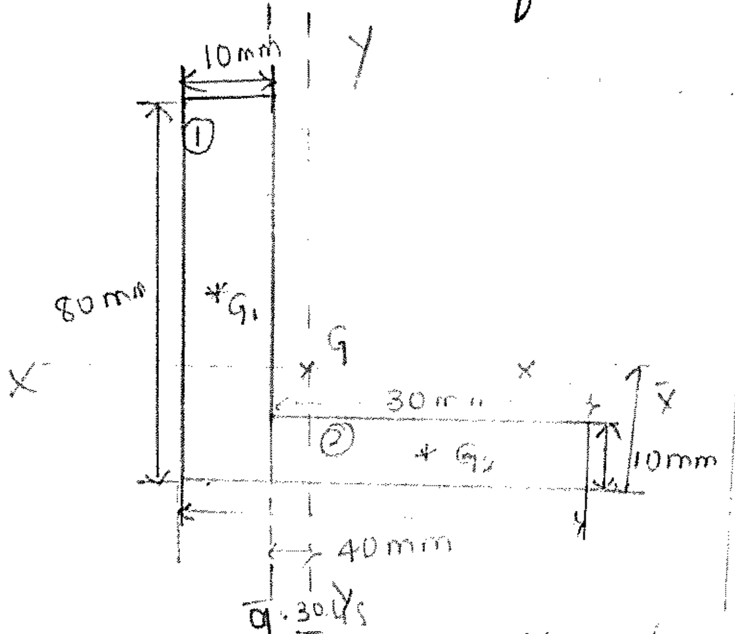


$a_1 = 200 \times 10$ $a_1 = 2000\ m^2$	$x_1 = 10/2$ $= 5$ mm
$a_2 = 90 \times 10$ $a_2 = 900\ m^2$	$x_2 = 10 + 45$ $= 55$ mm
$a_3 = 90 \times 10$ $a_3 = 900\ m^2$	$x_3 = 10 + 45$ $= 55$ mm
$\Sigma a = 3800\ m^2$	$\bar{x} = 28.68$

$$\bar{x} = \frac{\Sigma a . x}{\Sigma a} = \frac{(2000 \times 5) + (900 \times 55) + (900 \times 55)}{3800}$$

$$\boxed{\bar{x} = 28.68\ mm}$$

④ Locate Centroid of L-section shown in figure.

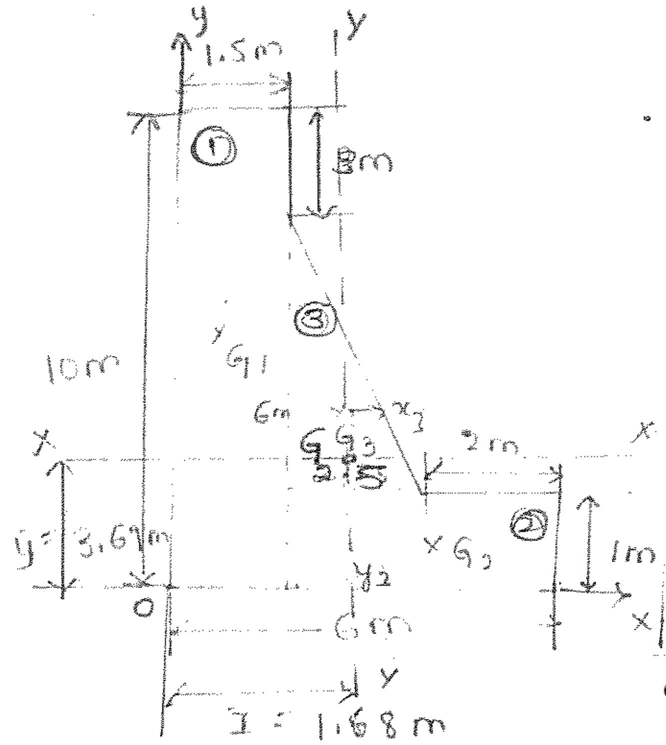


$a_1 = 80 \times 10$ $a_1 = 800\ m^2$	$x_1 = 10/2$ $x_1 = 5$ mm	$y_1 = 80/2$ $y = 40$ mm
$a_2 = 30 \times 10$ $a_2 = 300\ m^2$	$x_2 = 10 + 15$ $x_2 = 25$ m	$y_2 = 5$ m
$\Sigma a = 1100$	$\bar{X} = 10.45$ mm	$\bar{y} = 30.45$ mm

$$\bar{X} = \frac{\Sigma a . X}{\Sigma a} = \frac{(800 \times 5) + (300 \times 25)}{1100} = 10.45\ mm$$

$$\bar{Y} = \frac{\Sigma a\ Y}{\Sigma a} = \frac{(800 \times 40) + (300 \times 5)}{1100} = 30.45$$

Locate centroid of the dam section shown in the figure



$a_1 = 10 \times 1.5 \quad x_1 = 0.75\,mm \quad y_1 = 5\,m$

$a_1 = 15\,m^2$

$a_2 = 4.5 \times 1 \quad x_2 = 1.5 + \frac{4.5}{2} \quad y_2 = \frac{1}{2}$

$a_2 = 4.5\,m^2 \quad x_2 = 3.75\,m \quad y_2 = 0.5\,m$

$a_3 = \frac{1}{2} \times 6 \times 2.5 \quad x_3 = 1.5 + \frac{1}{3} \times 2.5 \quad y_3 = 1 + \frac{6}{3}$

$a_3 = 7.5\,m^2 \quad x_3 = 2.33\,m \quad y_3 = 3\,m$

$\Sigma a = 27$

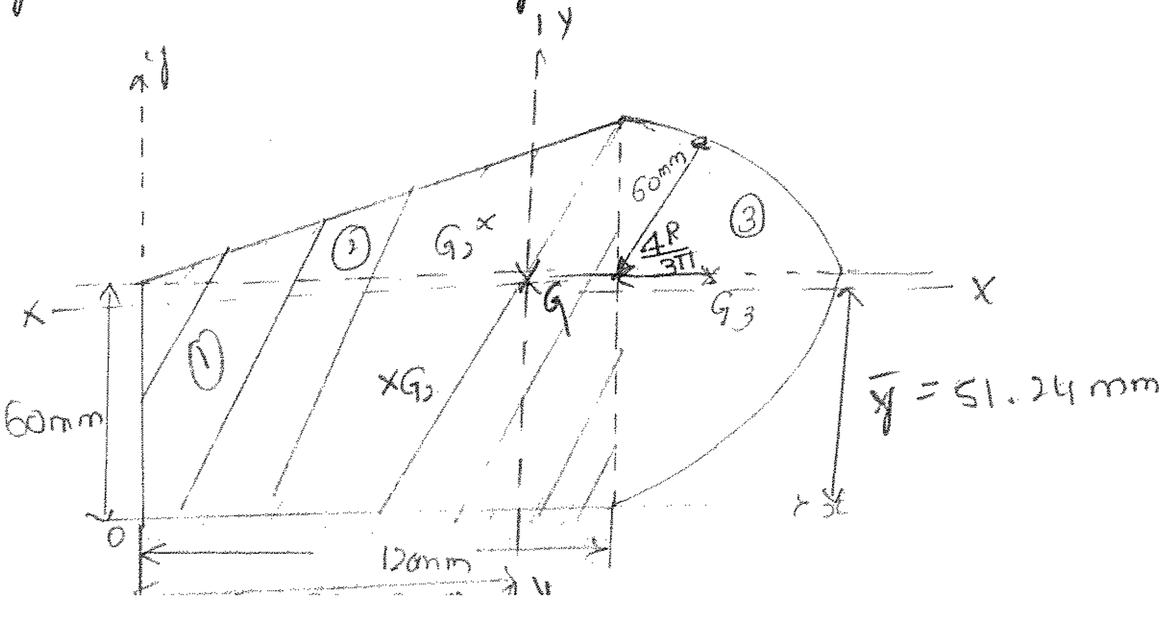
$\bar{x} = \frac{\Sigma a.x}{\Sigma a} = \frac{(15 \times 0.75) + (4.5 \times 3.75) + (7.5 \times 2.33)}{27}$

$\boxed{\bar{x} = 1.68\,m}$

$\bar{y} = \frac{\Sigma a.y}{\Sigma a} = \frac{(15 \times 5) + (4.5 \times 0.5) + (7.5 \times 3)}{27}$

$\boxed{\bar{y} = 3.69\,m}$

6] Locate Centroid of composite section shown in the figure. w.r.t OX & OY axes.



$a_1 = 120 \times 60$ $= 7200\,mm^2$	$x_1 = 120/2$ $= 60\,mm$	$y_1 = 60/2$ $= 30\,mm$
$a_2 = \frac{1}{2} \times 120 \times 60$ $= 3600\,mm^2$	$x_2 = \frac{2}{3} \times 120$ $= 80\,mm$	$y_2 = \frac{1}{3} \times 60 + 60$ $= 80\,mm$
$a_3 = \frac{\pi r^2}{2} = \frac{\pi 60^2}{2}$ $= 5652\,mm^2$	$x_3 = 120 + \frac{4R}{3\pi}$ $= 145.46\,mm$	$y_3 = 60\,mm$
$\Sigma a =$ $16,452\,mm^2$		

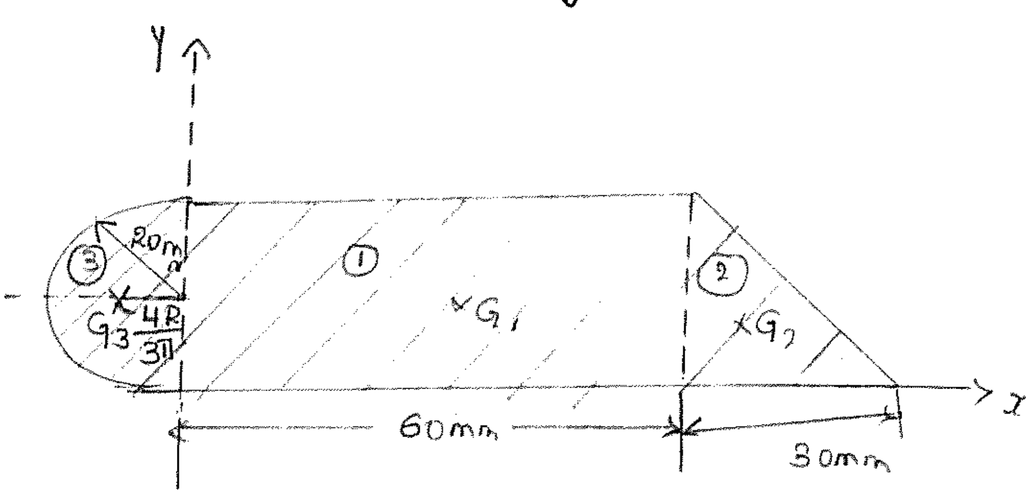
$$\bar{x} = \frac{\Sigma a.x}{\Sigma a} = \frac{(60 \times 7200) + (3600 \times 80) + (5652 \times 145.46)}{16452}$$

$$\bar{x} = 93.73\,mm$$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{(7200 \times 30) + (3600 \times 80) + (5652 \times 60)}{16452}$$

$$\bar{y} = 51.24\,mm$$

(7) Locate centroid of composite section shown in the figure w.r.to ox and oy axes.



$a_1 = 60 \times 40$ $= 2400 \text{ mm}^2$	$x_1 = 60/2$ $= 30 \text{ mm}$	$y_2 = 40/2$ $= 20 \text{ mm}$
$a_2 = \cancel{1/2} \times 30 \times \cancel{40}^{10}$ $= 600 \text{ m}^2$	$x_2 = \cancel{1/3} \times \cancel{30}^{10} + 60$ $= 70 \text{ mm}$	$y_2 = 1/3 \times 40$ $= 13.33 \text{ mm}$
$a_3 = \frac{\pi R^2}{2} = \frac{\pi \times 20^2}{2}$ 628.32 mm^2	$x_3 = \frac{-4(20)}{3\pi}$ $= -8.49 \text{ mm}$	$y_3 = 20 \text{ mm}$
$\Sigma a = 3628.32$		

$$\bar{x} = \frac{\Sigma a . x}{\Sigma a} = \frac{(2400 \times 30) + (600 \times 70) + (628.32 \times -8.49)}{3628.32}$$

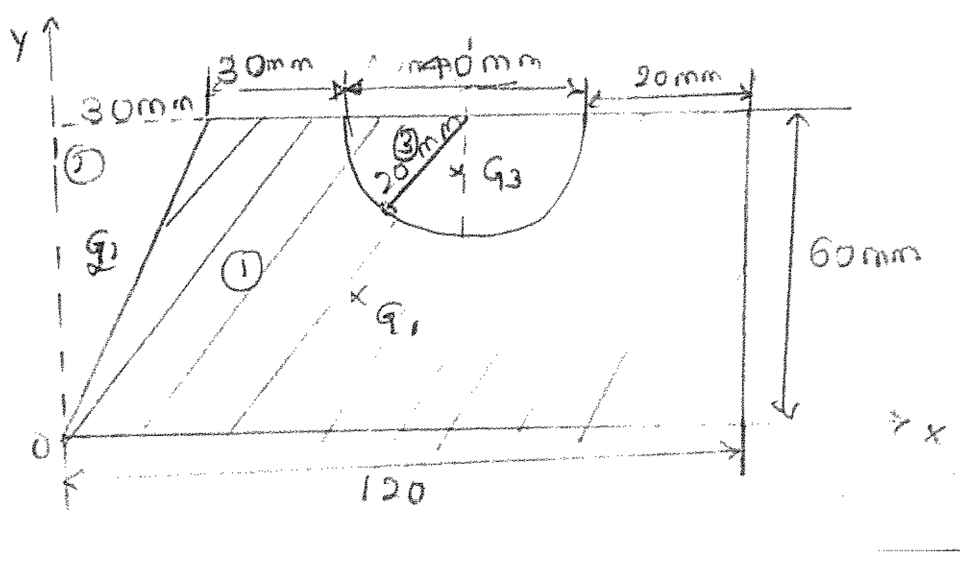
$$\bar{x} = \underline{29.95 \text{ mm}} \text{ from } oy$$

$$\bar{y} = \frac{\Sigma a . y}{\Sigma a} = \frac{(2400 \times 20) + (600 \times 13.33) + (628.32 \times 20)}{3628.32}$$

$$\bar{y} = \underline{18.89 \text{ mm}} \text{ from } ox.$$

Shaded area

8] Locate centroid of the shaded area shown in the figure. w.r. to ox & oy axes.



$a_1 = 120 \times 60$ $a_1 = 7200 mm^2$	$x_1 = 60mm$	$y_1 = 30mm$
$a_2 = -\frac{1}{2} \times 20 \times 60$ $= -600 \, mm^2$	$x_2 = \frac{1}{3} \times 30$ $x_2 = 10mm$	$y_2 = \frac{2}{3} \times 60$ $= 40mm$
$a_3 = -\frac{\pi (20)^2}{2}$ $a_3 = -628.32 \, mm^2$	$x_3 = 30 + 30 \times 2$ $= 80mm$	$y_3 = \left(60 - \frac{4(20)}{3\pi}\right.$ $y_3 = 51.51 mm$

$$\Sigma a = -628.32 - 600 + 7200$$

$$= 5971.68 \, mm^2$$

3.49

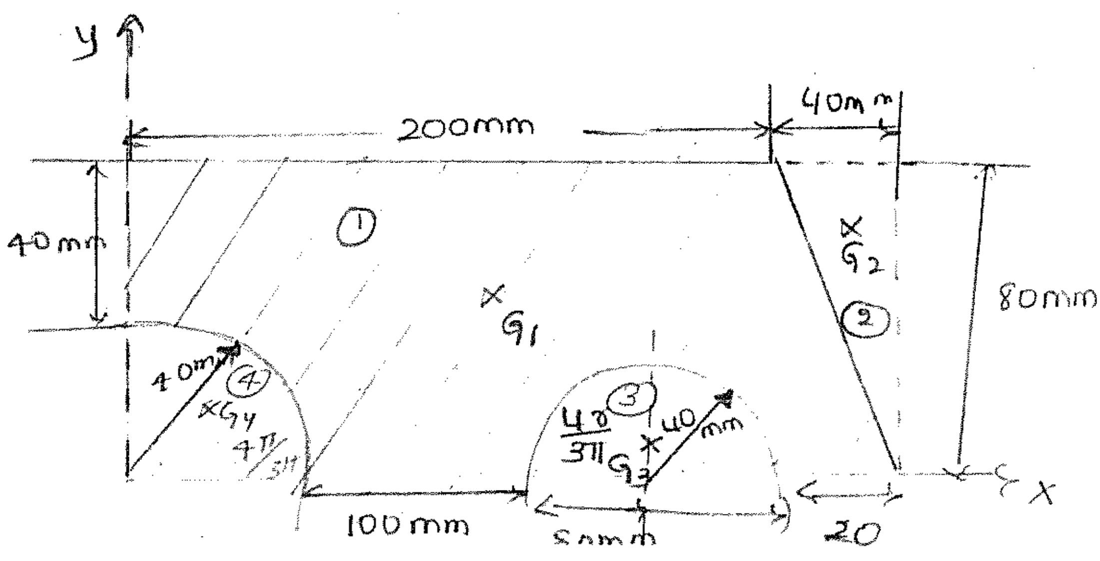
$$\bar{x} = \frac{\Sigma a \cdot x}{\Sigma a} = \frac{(7200 \times 60) + (-600 \times 10) + (-628.32 \times 80)}{5971.68}$$

$$\bar{x} = 65.71 mm \text{ from } oy.$$

$$\bar{y} = \frac{\Sigma a \cdot y}{\Sigma a} = \frac{(7200 \times 30) - (600 \times 40) - (628.32 \times 51.5)}{5971.68}$$

$$\bar{y} = 26.03 \, mm \text{ from } ox.$$

9] Locate centroid of the shaded area shown in the figure w.r to ox and oy axes.



$a_1 = 240 \times 80$ $= 19200 mm^2$	$x_1 = \frac{240}{2}$ $x_1 = 120 mm$	$y_1 = \frac{80}{2}$ $y_1 = 40 mm$
$a_2 = -\frac{1}{2} \times \cancel{80}^{40} \times 40$ $a_2 = -1600 mm^2$	$x_2 = 200 + \frac{2}{3} \times 40$ $x_2 = 226.67 mm$	$y_2 = \frac{2}{3} \times 80$ $y_2 = 53.33 mm$
$a_3 = \frac{-\pi r^2}{2} = \frac{-\pi (40)^2}{2}$ $a_3 = -2513.27 mm^2$	$x_3 = 40 + 40 + 100$ $x_3 = 180 mm$	$y_3 = 4(40)/3\pi$ $= 16.98 mm$
$a_4 = \frac{-\pi r^2}{4} = \frac{\pi (40)^2}{4}$ $a_4 = -1256.64 mm^2$	$x_4 = 4(40)/3\pi$ $= 16.98 mm$	$y_4 = 4(40)/3\pi$ $y_4 = 16.98 mm$
$\Sigma a = 13,830.09 mm^2$		

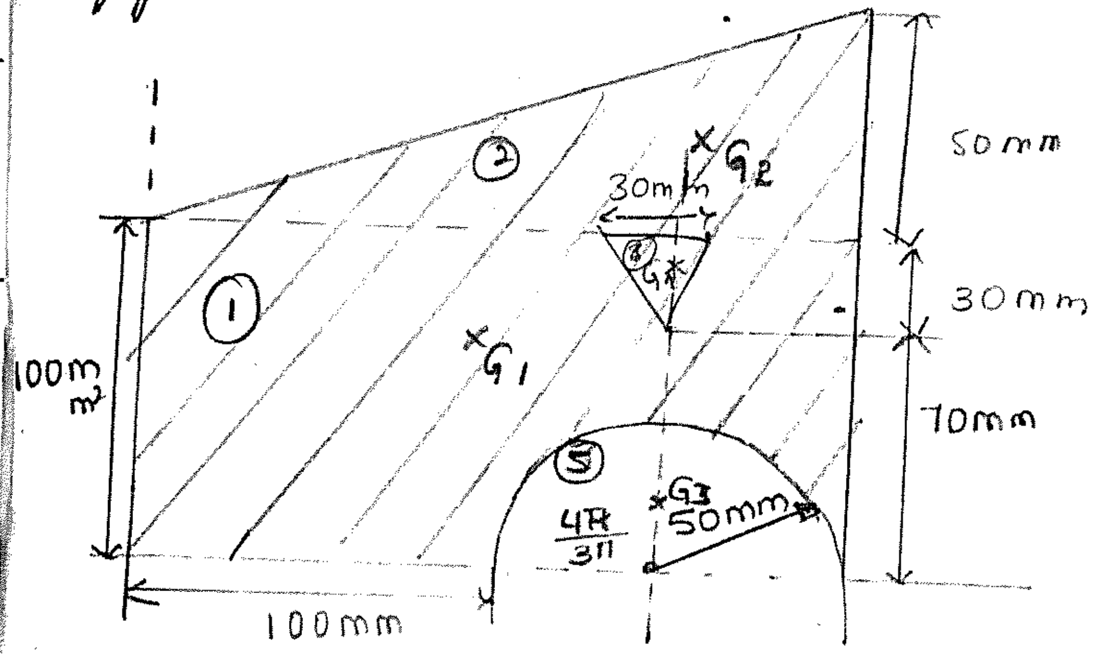
$$\bar{X} = \frac{\Sigma a.x}{\Sigma a} = \frac{(19200 \times 120) - (1600 \times 226.67) - (2513.27 \times 180) - (1256.64 \times 16.98)}{13,830.09}$$

$$\bar{X} = 106.12 mm$$

$$\bar{y} = \frac{\Sigma a.y}{\Sigma a} = \frac{(19200 \times 40) - (1600 \times 53.33) - (2513.27 \times 16.98) - (1256.64 \times 16.98)}{13,830.09}$$

$$\bar{y} = 44.73 mm$$

Q) Locate the Centroid of the Shaded area shown in the figure w.r. to Ox and Oy axes.



$a_1 = 200 \times 100$ $= 20000\,mm^2$	$x_1 = 100\,mm$	$y_1 = 50\,mm$
$a_2 = \frac{1}{2} \times 200 \times 50$ $a_2 = 5{,}000\,mm^2$	$x_2 = \frac{2}{3} \times 200$ $x_2 = 133.33\,mm$	$y_2 = \frac{1}{3} \times 50 + 100$ $y_2 = 116.66\,mm$
$a_3 = -\frac{4 \times 50}{3 \times \pi}$ $= -3927$	$x_3 = 100 + 50$ $= 150\,mm$	$y_3 = \frac{4(50)}{3\pi}$ $= 21.22\,mm$
$a_4 = -\frac{1}{2} \times 30 \times 30$ $a_4 = -450\,mm^2$	$x_4 = 100 + 50$ $= 150\,mm$	$y_4 = 70 + \frac{2}{3} \times 30$ $y_4 = 90\,mm$
$\Sigma a = 20623\,mm^2$		

$$\bar{x} = \frac{\Sigma a \cdot x}{\Sigma a} = \frac{(20{,}000 \times 100) + (5000 \times 133.33) - (3927 \times 150) - (450 \times 150)}{20623}$$

$\bar{x} = 97.46\,mm$

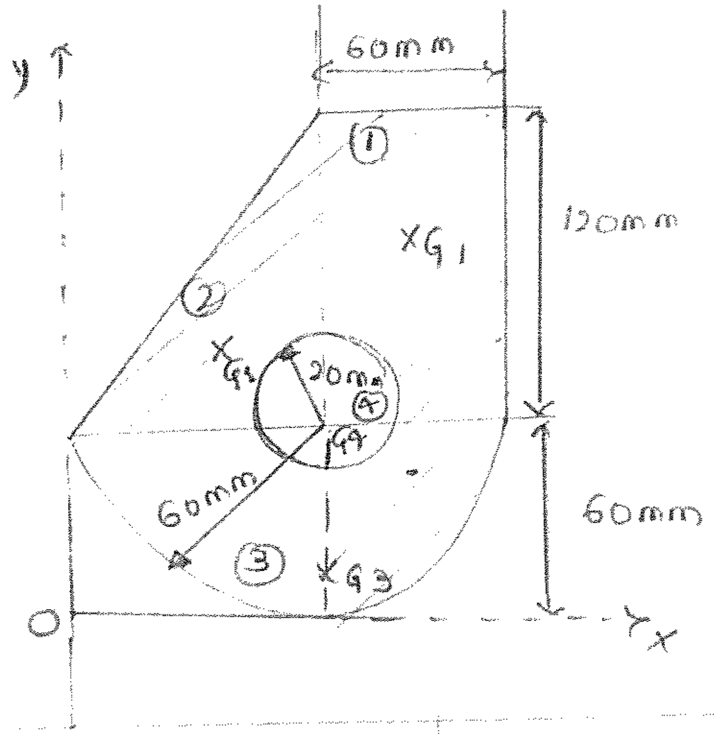
$\bar{y} = 70.77\,mm$

$$\boxed{\bar{x} = 97.46\,mm}$$

$$\bar{y} = \frac{\Sigma a \cdot y}{\Sigma a} = \frac{(20{,}000 \times 50) + (5000 \times 116.66) - (3927 \times 21.22) - (450 \times 90)}{20623}$$

$$\boxed{\bar{y} = 70.76\,mm}$$

(17) Locate the centroid of shaded area shown in the figure w.r.t. to OX and OY axes.



$a_1 = 60 \times 120$

$a_1 = 7200\,mm^2$

$x_1 = 60 + 30$

$x_1 = 90\,mm$

$y_1 = 60 + 60$

$= 120\,mm$

$a_2 = \frac{1}{2} \times 60 \times 120$

$= 3600\,mm^2$

$x_2 = \frac{2}{3} \times 60$

$x_2 = 40\,mm$

$y_2 = \frac{1}{3} \times 120 + 60$

$y_2 = 100\,mm$

$a_3 = \frac{\pi r^2}{2} = \frac{\pi (60)^2}{2}$

$a_3 = 5654.8\,mm^2$

$x_3 = 60\,mm$

$y_3 = 60 - \frac{4R}{3\pi}$ (60)

$= +34.53$

$a_4 = -\pi r^2 = \pi (20)^2$

$a_4 = -1256.64\,mm^2$

$x_4 = 60\,mm$

$y_4 = 60\,mm$

$\Sigma a = 15198.16\,mm^2$

$\bar{y} = \frac{\Sigma a \cdot y}{\Sigma a}$

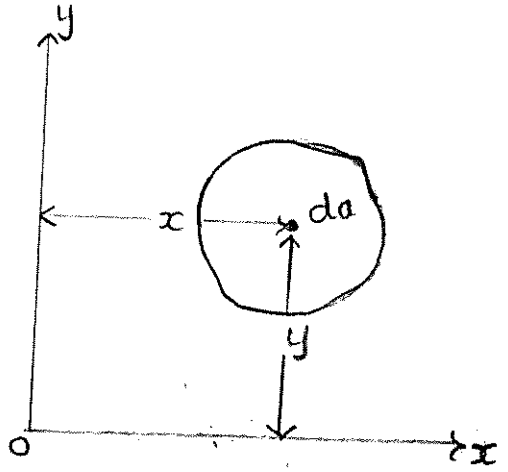
$\bar{y} = 88.42\,mm$

$$\bar{x} = \frac{\Sigma a \cdot x}{\Sigma a} = \frac{(7200 \times 90) + (3600 \times 40) + (5654.8 \times 60) - (1256.64 \times 60)}{15198.16}$$

$\bar{x} = 69.47\,mm$

MOMENT OF INERTIA OF PLANE AREAS

[OR Second moment of areas]



Consider the area shown in the figure. da is an elemental area with co-ordinates x and y.

The term $\Sigma da.y^2$ is called moment of inertia of the area about x axis and is denoted by I_{xx}.

Thus $\boxed{I_{xx} = \Sigma da.y^2}$ mm^4

$|||^{ly}$, the term $\Sigma da.x^2$ is called moment of inertia of the area about y axis and is denoted by I_{yy}.

$\boxed{I_{yy} = \Sigma da.x^2}$ mm^4

Thus, the MI of a plane area (or lamina) which has no mass is defined as the product of the area and the square of the distance between the area and the reference axis.

Rectangular MI :-

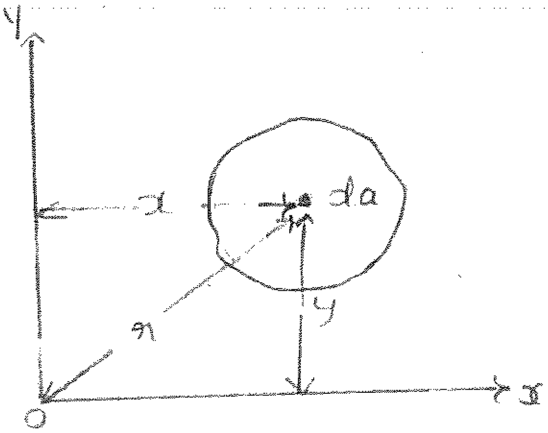
The MI of a plane area is also referred as rectangular MI when the reference axis lies in the plane of the area.

MI about ox axes = $I_{xx} = \Sigma da.y^2 = \int da.y^2$

MI about oy axis = $I_{yy} = \Sigma da.x^2 = \int da.x^2$

Polar moment of Intertia [I_p or I_{zz} or J] :-

Moment of intertia about an axis perpendicular to the plane of an area is known as polar M.I.



$\therefore I_p = I_{zz} = \Sigma da \cdot r^2 = \int da \cdot r^2$

Product of Inertia (I_{xy}) :-

It is defined as the product of the area and the two centroidal co-ordinates of the area with respect to the reference axis.

$$I_{xy} = \int da \cdot x \cdot y$$

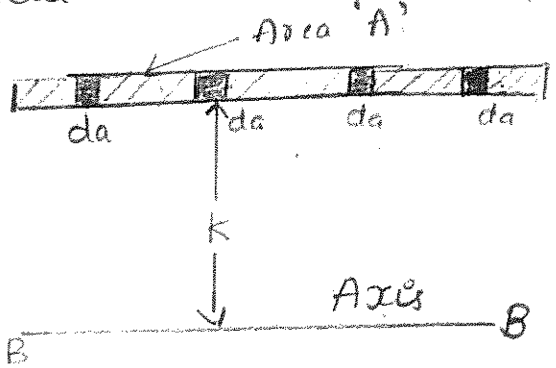
The product of inertia of an area w.r.t two rectangular axis is zero, if one of the axis is an axis of symmetry.

$\therefore$ The product of inertia is zero for symmetrical figures.

Radius of Gyration (K) :

$$I_{BB} = \Sigma da \cdot K^2 = K^2 \Sigma da$$

$$\therefore I_{BB} = AK^2$$



Here A = cross sectional area.

Radius of gyration is a mathematical term defined by the relation,

$$K = \sqrt{\frac{I}{A}} \qquad I = AK^2$$

$\therefore$ Radius of gyration (K) is that distance whose square when multiplied by the area gives the M.I of that area.

Thus, we can have,

$K_{xx} = \sqrt{\frac{I_{xx}}{A}}$ mm

$K_{AB} = \sqrt{\frac{I_{AB}}{A}}$, $K_{yy} = \sqrt{\frac{I_{yy}}{A}}$ mm

The polar Radius of gyration, K_{zz} (or K_p) $= \sqrt{\frac{I_{zz}}{A}}$

Where I_{zz} (or I_p) is the polar MI of that area

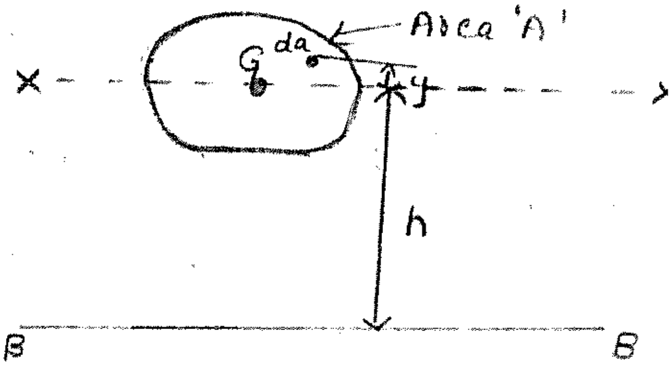
Here, $I_{zz} = I_{xx} + I_{yy}$.

Theorems of Moments of Interia :-

** 1] Parallel axis Theorem :-

Moment of inertia about any axis in the plane of an area is equal to the sum of MI about parallel centroidal axis and the product of area and square of the distance between the two parallel axes.

From the figure, the above theorem means :



$I_{BB} = I_{GG} + Ah^2$

Where,

I_{BB} = I_{GG} + MI about the axis BB.

I_{GG} = MI about ~~the axis~~ BB. centrodal axis parallel to BB ($= I_{xx}$)

A = the area of the plane figure.

h = Distance between the axis BB and the parallel centroidal axis GG (axis XX)

Proof :-

$$I_{BB} = \int da\,(h+y)^2$$

$$= \int da\,(h^2+y^2+2hy)$$

$= \int da \cdot h^2 + \int da \cdot y^2 + \int da\, 2hy.$

$= h^2 \int da + \int da \cdot y^2 + 2h \int da \cdot y.$

Here $\int da = A$. $\int da\, y^2 = I_{xx}$ and $\int da \cdot y = 0$
since for the whole area, the value of $y = 0$

$\therefore \quad I_{BB} = Ah^2 + I_{xx}$

$\therefore \boxed{I_{BB} = I_{GG} + Ah^2}$

Here $I_{xx} = I_{GG} = MI$
w.r.t centroidal axis
parallel to axis BB.

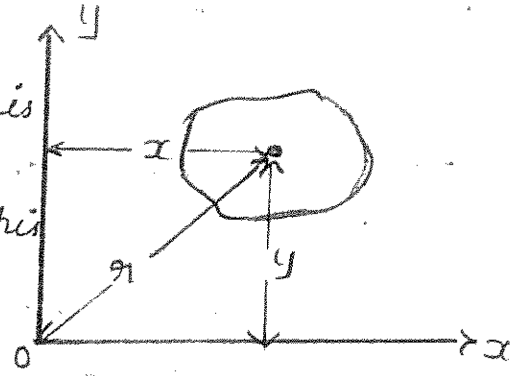
The above eq^n cannot be applied to any two parallel axis. One of the axis (GG) must be centrodial axis only.

2] Perpendicular Axis Theorem :- Important

The moment of inertia of an area about an axis perpendicular to its plane (polar MI) at any point 'O' is equal to the sum of moments of inertia about any two mutually perpendicular axis through the same point and lying in the plane of the area.

Refering to the figure, if z-z is the axes normal to the plane of paper passing through O, as per this theorem,

$I_{zz} = I_{xx} + I_{yy}.$



Proof :-

$$I_{zz} = \int da \cdot r^2$$

$$= \int da (x^2 + y^2)$$

$$= \int da \cdot x^2 + \int da \cdot y^2$$

$$I_{zz} = I_{yy} + I_{xx}$$

$\because \int da \cdot x^2 = I_{yy}$

$\int da \cdot y^2 = I_{xx}$

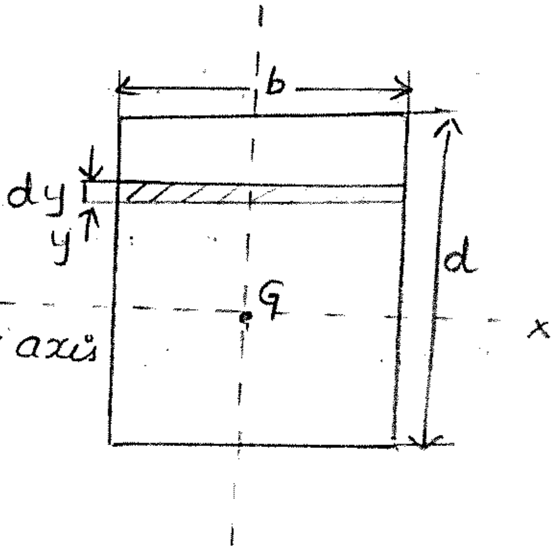
$\therefore \boxed{I_{zz} = I_{xx} + I_{yy}}$

The polar MI of an area w.r.t any axis is equ
to the sum of the moments of inertia of the area w.r.t any two rectangular axes in the plane of the area that intersects with the given polar axis.

Moment of Inertia of simple figures (or areas) from method of integration or from first principles.

MI of Rectangle about the centroidal axis :-

Consider an elemental strip of thickness dy at a distance y from the axis XX.



Area of the element, $da = b \cdot dy$

MI of the element about the XX axis

$$= da \cdot y^2$$
$$= [b \cdot dy] y^2$$
$$= b \cdot y^2 \cdot dy$$

$\therefore$ The total MI of the rectangle about the centroidal axis, XX,

$$I_{xx} = \int_{-d/2}^{d/2} b \cdot y^2 \cdot dy$$

$$= b\left[\frac{y^3}{3}\right]_{-d/2}^{d/2}$$

$$= b\left[\frac{d^3}{24} + \frac{d^3}{24}\right] = b\left[\frac{2d^3}{24}\right]$$

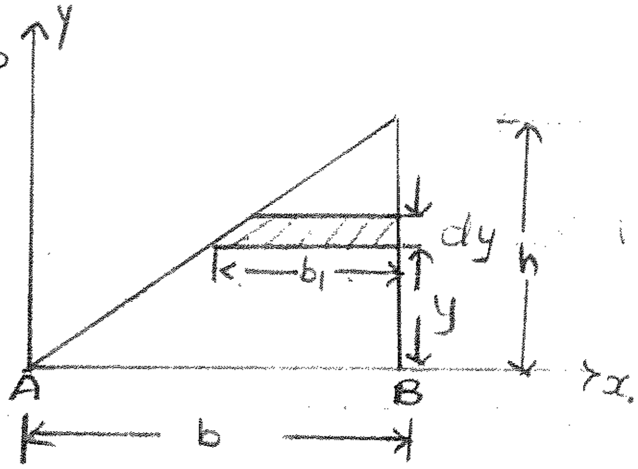
$$\boxed{I_{xx} = \frac{bd^3}{12}}$$

III^{ly}, MI about the centroidal axis YY parallel to the depth, $I_{YY} = \frac{bd^3}{12}$

M.I of a triangle about its base :-

Consider an elemental strip at a distance y from the base AB.

Let dy be the thickness of the strip and da its area. width of the strip is given by ;



From similar Δ^{ls}, $\frac{b_1}{b} = \frac{(h-y)}{h}$

$$\therefore b_1 = \frac{(h-y)}{h} \times b$$

Moment of Inertia this strip about base $AB = da.y^2$

$$= (b_1 . dy)y^2$$

$$= \left[\frac{(h-y)}{h} \times b \times dy\right] y^2$$

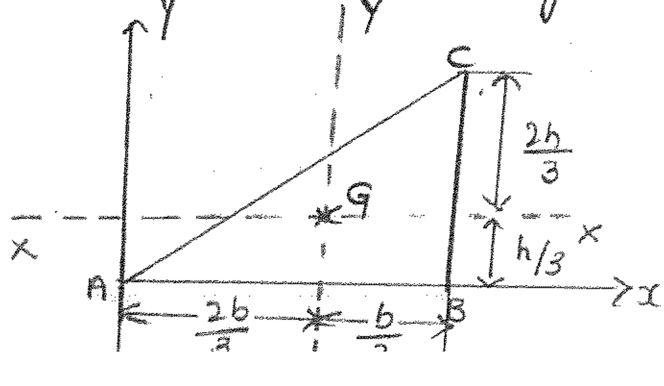
$\therefore$ Moment of Inertia of the triangle about base AB,

$$I_{AB} = \int_0^h \left[\frac{(h-y)}{h} \times b \times dy\right] y^2 = \int_0^h \left[y^2 - \frac{y^3}{h}\right] b\,dy$$

$$= b\left[\frac{y^3}{3} - \frac{y^4}{4h}\right]_0^h = b\left[\frac{h^3}{3} - \frac{h^4}{4h}\right]$$

$$\therefore I_{AB} = b\left[\frac{4h^4 - 3h^4}{12h}\right] = \frac{bh^3}{12}$$

MI of triangle about Centroidal axis XX, using parallel axis theorem :-



$$I_{AB} = I_{xx} + \left(\tfrac{1}{2} bh\right)\left(\frac{h}{3}\right)^2$$

$$\therefore \frac{bh^3}{12} = I_{xx} + \frac{bh^3}{18}$$

$$\therefore I_{xx} = \frac{bh^3}{12} - \frac{bh^3}{}$$

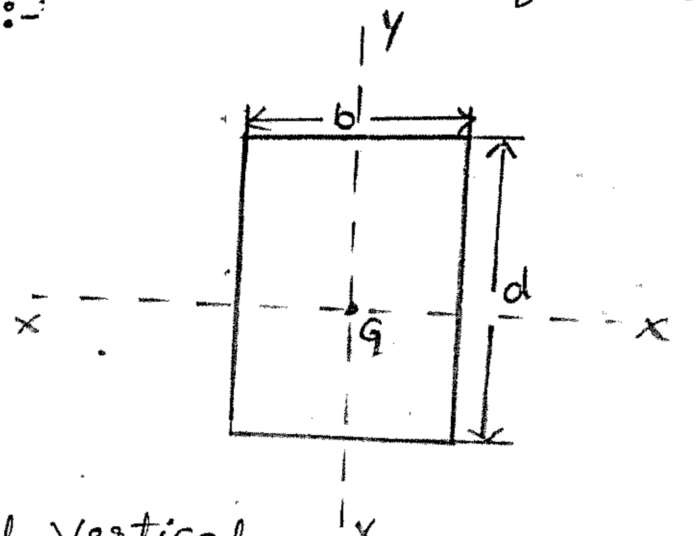
$$= \frac{3bh^3 - 2bh^3}{36}$$

$$\boxed{I_{xx} = \frac{bh^3}{36}}$$

III^{ly} $I_{BC} = \frac{hb^3}{12}$ and $\boxed{I_{yy} = \frac{hb^3}{36}}$

Moment of Interia [or Second moment of areas] of plane figures [or areas] :-

a] Rectangle :



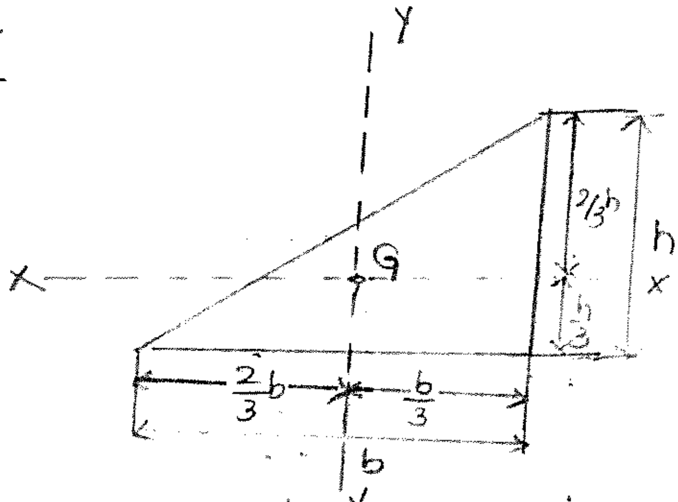
MI about centroidal Horizontal axis,

$$\boxed{I_{xx} = \frac{bd^3}{12}}$$

MI about centroidal vertical axis,

$$\boxed{I_{yy} = \frac{db^3}{12}}$$

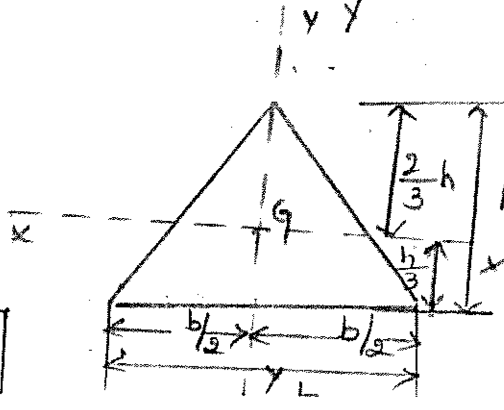
b] Right angled triangle :-



$$\boxed{I_{xx} = \frac{bh^3}{36}}$$

$$\boxed{I_{yy} = \frac{hb^3}{36}}$$

c] Symmetric triangle :-



$$\boxed{I_{xx} = \frac{bh^3}{36}}$$

$$I_{yy} = 2\left[\frac{h\left(\frac{b}{2}\right)^3}{12}\right]$$

$$= 2\left[\frac{hb^3}{96}\right] \quad \boxed{I_{yy} = \frac{hb^3}{48}}$$

d] Circle :-

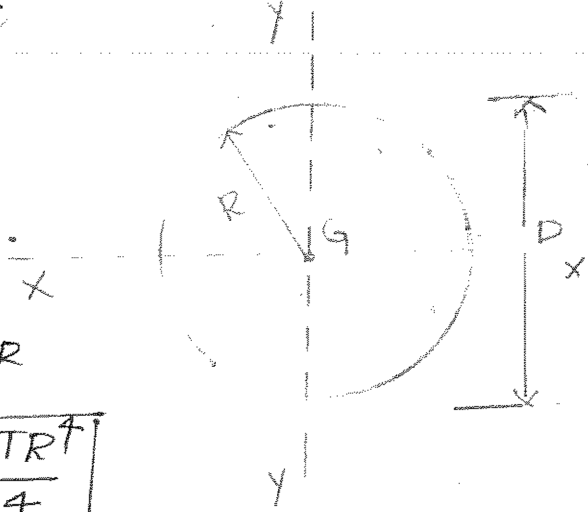
$$\boxed{I_{xx} = \frac{\pi D^4}{64}}$$

$$\boxed{I_{yy} = \frac{\pi D^4}{64}}$$

centre of the circle is centroid of the circle

Here $D = 2R$

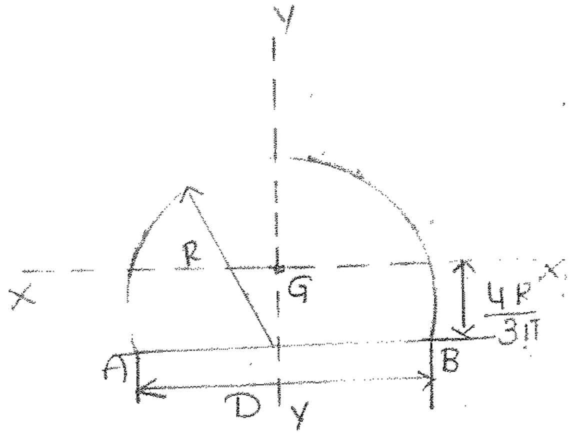
$$\therefore \boxed{I_{xx} = \frac{\pi R^4}{4}} \qquad \boxed{I_{yy} = \frac{\pi R^4}{4}}$$



e] Semicircle :-

$$\boxed{I_{AB} = \frac{1}{2}\left[\frac{\pi D^4}{64}\right]}$$

$$\boxed{I_{AB} = \frac{1}{2}\left[\frac{\pi R^4}{4}\right]}$$



From parallel axis theorem,

$$I_{AB} = I_{xx} + a\left(\frac{4R}{3\pi}\right)^2$$

$$\frac{\pi R^4}{8} = I_{xx} + \left(\frac{\pi R^2}{2}\right)\left(\frac{4R}{3\pi}\right)^2$$

$$\boxed{I_{xx} = 0.11 R^4}$$

$$\boxed{I_{yy} = \frac{1}{2}\left[\frac{\pi D^4}{64}\right]}$$

$$\boxed{I_{yy} = \frac{1}{2}\left[\frac{\pi R^4}{4}\right]}$$

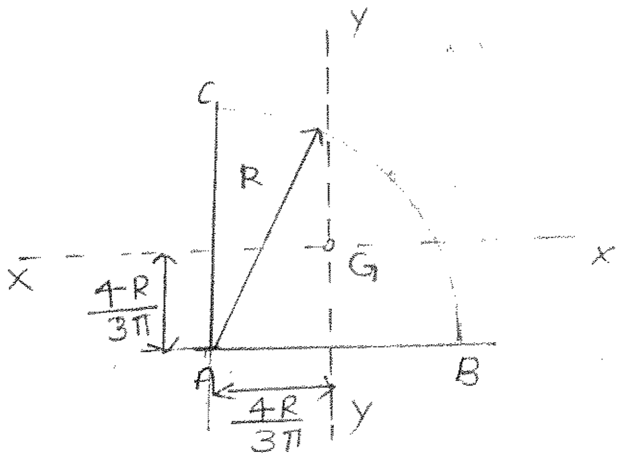
f] Quarter Circle :-

$$I_{AB} = \frac{1}{4}\left[\frac{\pi D^4}{64}\right]$$

$$I_{AB} = \frac{1}{4}\left[\frac{\pi R^4}{4}\right]$$

$$I_{AC} = \frac{1}{4}\left[\frac{\pi D^4}{64}\right]$$

$$I_{AC} = \frac{1}{4}\left[\frac{\pi R^4}{4}\right]$$



From parallel axis theorem

$$I_{AB} = I_{xx} + a\left(\frac{4R}{3\pi}\right)^2$$

$$\frac{\pi R^4}{16} = I_{xx} + \left(\frac{\pi R^2}{4}\right)\left(\frac{4R}{3\pi}\right)^2$$

$$\boxed{I_{xx} = 0.055 R^4}$$

$$\boxed{I_{yy} = 0.055 R^4}$$

(2) Diff □ MI polar MI.

(3). Diff. R.O gyration (K)

(4) State and explain perpendicular

(5) obtain an experiment of MI

$$Ixx = \frac{db^2}{12}$$

(6) Passing through the centroid. semicircle. and Quarter circle.

1st

$\frac{bd^3}{12}$

$\frac{db^3}{12}$

Problems:-

For the T-section shown below

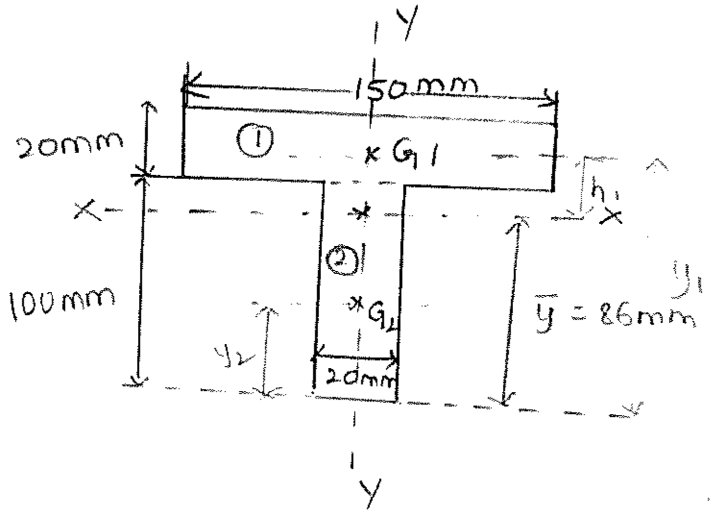
(i) M.I about centroidal horizontal axis (I_{xx})

(ii) Radius of gyration about centroidal horizontal axis (K_x

(iii) M.I about centroidal of vertical axis (I_{yy})

(iv) Radius of gyration about centroidal vertical axis (K_y

(v) Polar M.I (I_{zz})



$a_1 = 150 \times 20$ $\qquad$ $y_1 = 110\,mm$

$a_1 = \underline{3000\,mm^2}$

$a_2 = 100 \times 20$ $\qquad$ $y_2 = \underline{50\,mm}$

$= \underline{2000\,mm^2}$

$\bar{Y} = \frac{\Sigma a y}{\Sigma a}$

$\Sigma a = \underline{5000\,mm^2}$

$\boxed{\bar{y} = 86\,mm}$ $\quad y - \bar{y}$ $\quad$ difference $\quad y = \bar{y}$

$h_1 = y_1 - \bar{y}$ $\qquad h_1 = y_1 \sim \bar{y}$

(i) $I_{xx} = \left[\frac{b_1 d_1^3}{12} \right]$ vertical axis

$I_{xx} = \left[\frac{150 \times 20^3}{12} + (150 \times 20)(110 - 86)^2 \right] +$

$\left[\frac{20 \times 100^3}{12} + (20 \times 100)\;(86 - 50)^2 \right]$ $\qquad h_2 = y_2 \sim \bar{y}$ $\quad \bar{y} = y_2$

$I_{xx} = \underline{6.08 \times 10^6\,mm^4}$

(ii) $I_{xx} = A K^2_{xx}$

$= \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{6.08 \times 10^6}{5000}} = 34.87\,mm$

iii) $I_{yy} = \left[\frac{20 \times 150^3}{12}\right] + \left[\frac{100^3 \times 20}{12}\right]$

$I_{yy} = \underline{5.69 \times 10^6\ mm^4}$

iv) $I_{yy} = A K_{yy}^2$

$K_{yy} = \sqrt{\frac{I_{yy}}{A}}$

$= \sqrt{\frac{5.69 \times 10^6}{5000}}$

$K_{yy} = \underline{33.73\ mm}$

(v) I_{zz} or $I_p = I_{xx} + I_{yy}$

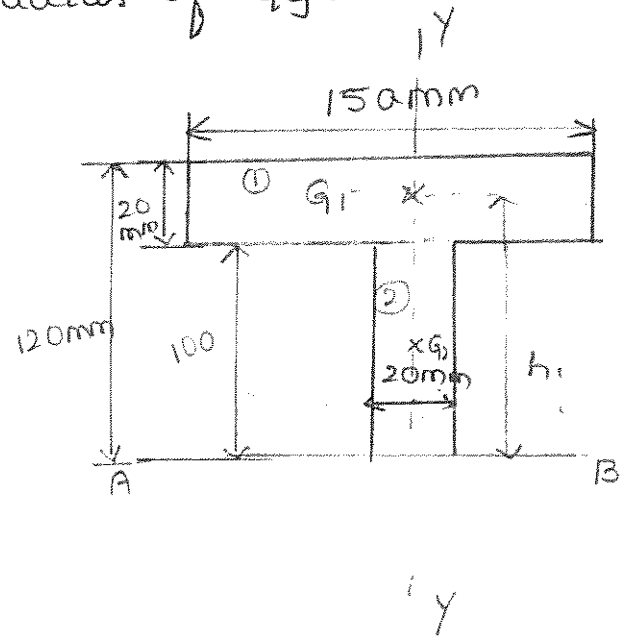
$= 6.08 \times 10^6 + 5.69 \times 10^6$

$= \underline{11.77 \times 10^6\ mm^4}$

2] For the T-Section shown below find,

(i) M I about the axes AB (I_{AB})

(ii) Radius of Gyration about axes AB. (K_{AB})



$I_{AB} = \left[\frac{150 \times 20^3}{12} + (150 \times 20)(110)^2\right] + \left[\frac{20 \times 100^3}{12} + (100 \times 20)(50)^2\right]$

$\boxed{I_{AB} = 4.307 \times 10^7\ mm^4}$

(ii) $K_{AB} = ?$ $I_{AB} = A K_{AB}^2$

$$K_{AB} = \sqrt{\frac{I_{AB}}{A}}$$

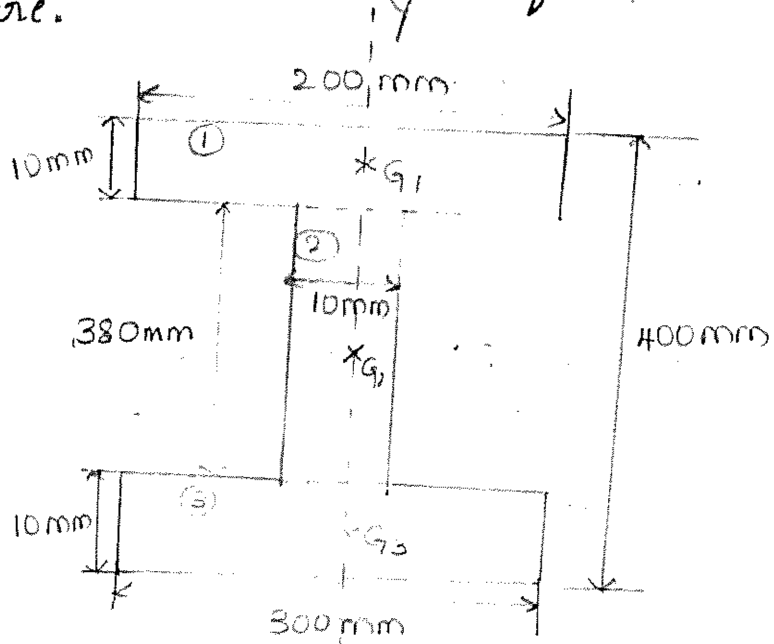
$A = (150 \times 20) + (100 \times 20)$

$= \underline{\underline{5000\,mm^2}}$

$$K_{AB} = \sqrt{\frac{4.307 \times 10^7}{5000}}$$

$$\boxed{K_{AB} = 92.81\,mm}$$

③ Determine Polar MI of I Section shown in the figure.



$a_1 = 200 \times 10$ $\quad y_1 = 395\,mm$

$a_1 = 2000\,mm^2$

$a_2 = 10 \times 380$ $\quad y_2 = 200\,mm$

$a_2 = 3800\,mm^2$

$a_3 = 10 \times 300$ $\quad y_3 = 5\,mm$

$a_3 = 3000\,mm^2$

$\Sigma a = 88{,}00\cancel{0}\,mm^2$

$$\bar{y} = \frac{\Sigma a \cdot y}{\Sigma a} = \frac{(2000 \times 395) + (3800 \times 200) + (3000 \times 15)}{88000}$$

$$\boxed{\bar{y} = 177.8\,mm}$$

$$I_{xx} = \left[\frac{200 \times 10^3}{12} + (200 \times 10)(395 - 177.8)^2\right] +$$

$$\left[\frac{10 \times 380^3}{12} + (380 \times 10)(177.8 - 200)^2\right] +$$

$$\left[\frac{300 \times 10^3}{12} + (300 \times 10)(177.8 - 5)^2\right]$$

$$= 0.9 \times 10^8 + 0.46 \times 10^8 + 0.9 \times 10^8$$

$$= \underline{2.3 \times 10^8 \text{ mm}^4}$$

$$I_{yy} = \left[\frac{10 \times 200^3}{12} + 0\right] + \left[\frac{380 \times 10^3}{12} + 0\right] + \left[\frac{10 \times 300^3}{12} + 0\right]$$

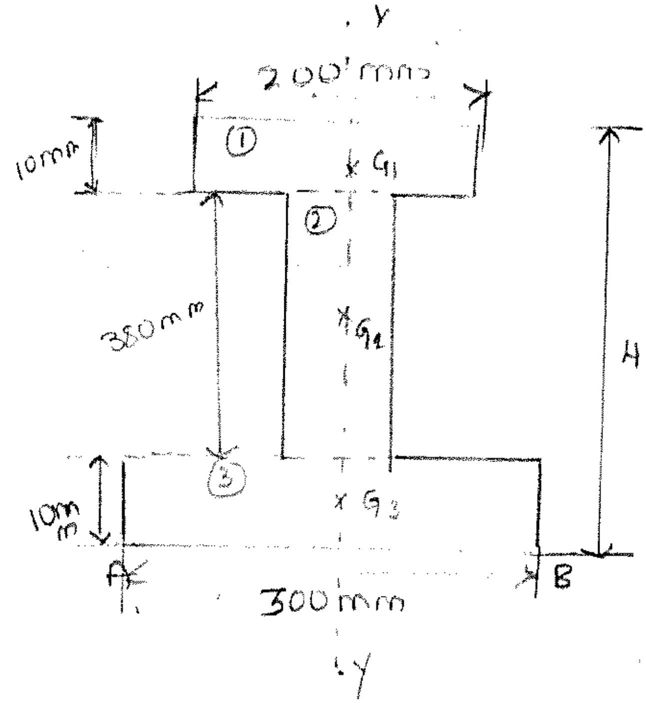
$$= \underline{0.292 \times 10^8 \text{ mm}^4}$$

$$I_{zz} = I_{xx} + I_{yy}$$

$$= 2.3 \times 10^8 + 0.292 \times 10^8$$

$$= \underline{\underline{2.608 \times 10^8 \text{ mm}^4}}$$

(4) For the I-Section shown below find radius of gyratio about the axis AB.



$I_{AB} = A K_{AB}^2$

$K_{AB} = \sqrt{\dfrac{I_{AB}}{A}}$

$I_A = 5.099 \times 10^8$ mm

$A = (200 \times 10) + (380 \times 10) + (300 \times 10)$

$A = 8800 \text{ mm}^2$

$$I_{AB} = \left[\frac{200 \times 10^3}{12} + (200 \times 10) + 395^2\right] + \left[\frac{10 \times 380^3}{12} + 10 \times 38 \times 200\right] + \left[\frac{300 \times 10^3}{12} + (300 \times 10) \times 5\right]$$

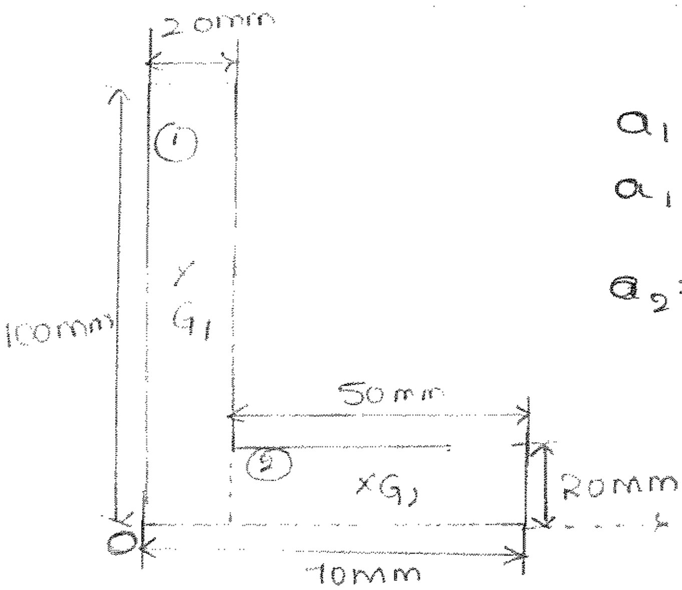
$$= 5.091 \times 10^8 \text{ mm}^4$$

$$K_{AB} = \sqrt{\frac{I_{AB}}{A}} = \sqrt{\frac{5.091 \times 10^8}{8800}} = 240.5 \text{ mm}$$

Find the L-section shown below find (i) M I about centroidal horizontal and vertical axis.

(i)

(ii) radii of gyration about centroidal horizontal and vertical axis (iii) polar M.I



$a_1 = 100 \times 20 \quad x_1 = 10mm \quad y_1 = 50mm$

$a_1 = 2000 mm^2$

$a_2 = 50 \times 20 \quad x_2 = 20+25 \quad y_2 = 10 mm$

$= 1000 mm^2 \quad = 45 mm$

$\Sigma a = 3000 mm^2$

$$\bar{X} = \frac{\Sigma a . x}{\Sigma a} = \frac{(2000 \times 10) + (1000 \times 45)}{3000}$$

$$\bar{X} = 21.67 \ mm$$

$$\bar{y} = \frac{\Sigma a . y}{\Sigma a} = \frac{(2000 \times 45) + (1000 \times 10)}{3000}$$

$$\bar{y} = 36.67 \ mm$$

$$I_{xx} = \left[\frac{20 \times 100^3}{12} + (20 \times 100)\ (50 - 36.67)^2 \right] + \left[\frac{50 \times 20^3}{12} + (50 \times 20)\ (36.62 - 10)^2 \right]$$

$$I_{xx} = 2.76 \times 10^6 \ mm^4$$

$$I_{xx} = A K_{xx}^2$$

$$K_{xx} = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{2.76 \times 10^6}{3000}}$$

$$K_{xx} = 30.33 \ mm$$

$$I_{yy} = \left[\frac{100 \times 20}{12} + (100 \times 20)(21.67 - 10)^2\right] +$$

$$\left[\frac{20 \times 50^3}{12} + (20 \times 50)(45 - 21.67)^2\right]$$

$$I_{yy} = \underline{\underline{1.09 \times 10^6 \text{ mm}^4}}$$

$$K_{yy} = \sqrt{\frac{I_{yy}}{A}} = \sqrt{\frac{1.09 \times 10^6}{3000}}$$

$$K_{yy} = \underline{\underline{19.06 \text{ mm}}}$$

$$I_{zz} = I_{xx} + I_{yy}$$

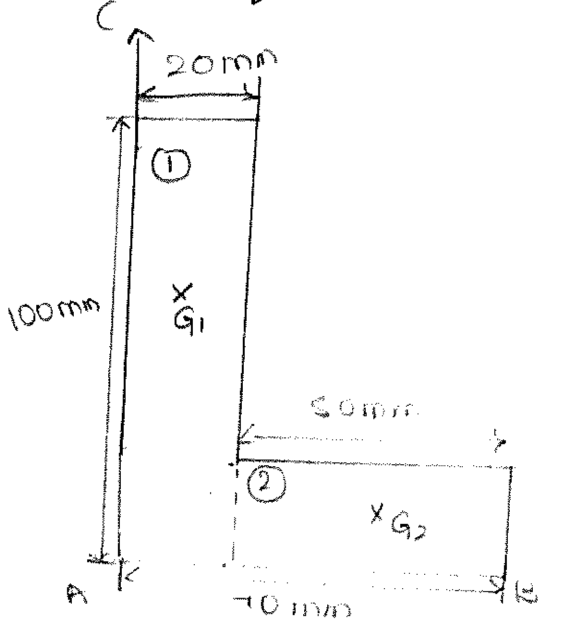
$$= 2.76 \times 10^6 + 1.09 \times 10^6$$

$$I_{zz} = \underline{\underline{3.85 \times 10^6 \text{ mm}^4}}$$

6] For the L-section shown below, find MI about

(i) AB and AC axes (I_{AB} & I_{AC})

(ii) Radii of gyration about AB & AC (K_{AB} & K_{AC})



$\Sigma a = 3000$ mm

$$I_{AB} = \left[\frac{20 \times 100^3}{12} + (20 \times 100)(50)^2\right]$$

$$+ \left[\frac{50 \times 20^3}{12} + (50 \times 20)(10)^2\right]$$

$$I_{AB} = 6. \quad \times 10^8 \text{ mm.}$$

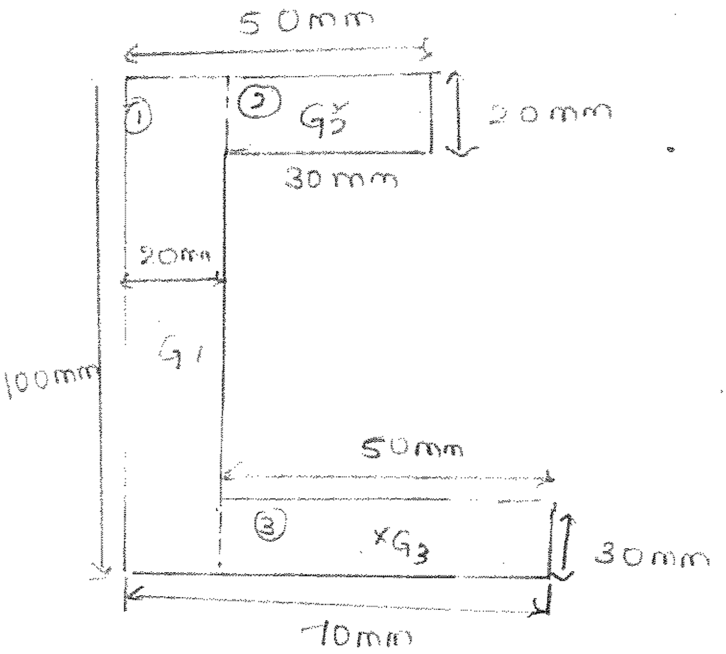
$$I_{AC} = \left[\frac{100 \times 20^3}{12} + (100 \times 20)(10\right.$$

$$+ \left[\frac{20 \times 50^3}{12} + (20 \times 5)^3 \, 45^2\right]$$

$$= \underline{\underline{2.5 \times 10^6 \text{ mm}^4}}$$

$$K_{AC} = \sqrt{\frac{I_{AC}}{n}} = \sqrt{\frac{2.5 \times 10^6}{3000}} = \underline{\underline{28.87\ mm}}$$

(7)



$a_1 = 20 \times 100$ $\quad x_1 = 10mm \quad y_1 = 50mm$

$= 2000\ mm^2$

$a_2 = 20 \times 30$ $\quad x_2 = 35mm \quad y_2 = 90mm$

$= 600\ mm^2$

$a_3 = 30 \times 50$ $\quad x_3 = 45\ mm \quad y_3 = 15\ mm$

$= 1500\ mm^2$

$$\Sigma a = \underline{\underline{4100\ mm^2}}$$

$$\bar{x} = \frac{\Sigma a.x}{\Sigma a} = \frac{(2000 \times 10) + (600 \times 35) + (1500 \times 45)}{4100}$$

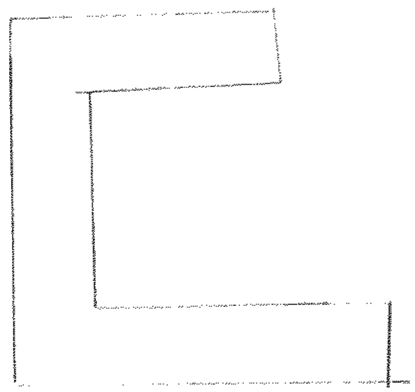
$$\bar{x} = \underline{\underline{26.46\ mm^4}}$$

$$\bar{y} = \frac{\Sigma a.y}{\Sigma a} = \frac{(2000 \times 50) + (600 \times 90) + (1500 \times 15)}{4100}$$

$\bar{y} =$ [illegible] mm

$$I_{xx} = \left[\frac{20 \times 100^3}{12} + 20 \times 100 \times 6.96^2 \right.$$

8] Determine radius of gyration about AB and AC.



$A = 4100\,mm^2$

$$I_{AB} = \left[\frac{20 \times 100^3}{12} + (20 \times 100)\,50^2\right] + \left[\frac{20 \times 30^3}{12} + 20 \times 30 \times 90^2\right] + \left[\frac{30 \times 50^3}{12} + 30 \times 50 \times 15^2\right]$$

$$I_{AB} = 6.6 \times 10^6 + 4.90 \times 10^6 +$$

$$I_{AB} = 12.15 \times 10^6\,mm$$

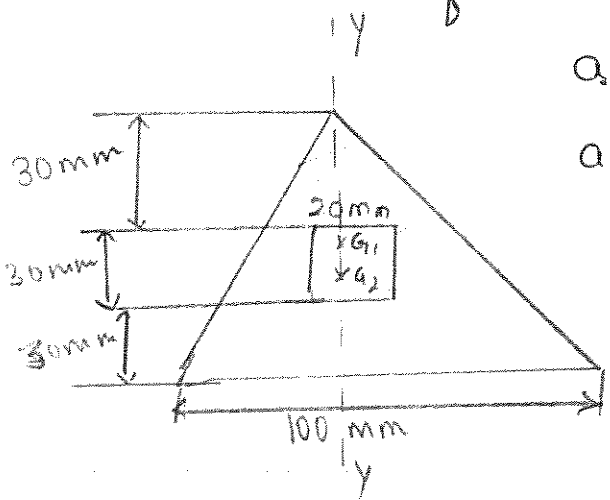
$$K_{AB} = \sqrt{\frac{I_{AB}}{A}} = \sqrt{\frac{12.1 \times 10^6}{4100}} = 54.32\,mm$$

$$I_{AC} = \left[\frac{20 \times 100^3}{12} + 20 \times 100 \times 10^2\right] + \left[\frac{20 \times 30^3}{12} + 20 \times 30 \times 35^2\right] + \left[\frac{30 \times 50^3}{12} + 30 \times 50 \times 45^2\right]$$

$I_{AC} =$

$$K_{AC} = \sqrt{\frac{I_{AC}}{A}} =$$

9] Find Polar M.I of shaded area shown in the figure.



$a_1 = \frac{1}{2} \times 100 \times 90$ $\quad x_1 = 50\,mm \quad y_1 = \frac{1}{3} \times 90 = 30\,mm$

$a_1 = 4500\,mm^2$

$a_2 = -20 \times 30 = -600\,mm^2$ $\quad y_2 = 30 + 15 = 45\,mm$

$\Sigma a = 3900.$

$$\bar{y} = \frac{\Sigma a.y}{\Sigma a} = \frac{\ldots}{3900}$$

$$\bar{y} = 27.69 \text{ mm}$$

$$I_{xx} = \left[\frac{100 \times 90^3}{30} + \tfrac{1}{2} \times 100 \times 90 \, (30 - 27.69)^2 \right] -$$

$$\left[\frac{20 \times 30^3}{12} + (20 \times 30) \, (45 - 27.69)^2 \right]$$

~~$I_{xx} = 18.24.$~~

$$I_{xx} = 1.8 \times 10^6 \text{ mm}^4$$

$$I_{yy} = \left[\frac{90 \times 100^3}{48} \right] - \left[\frac{30 \times 20^3}{12} \right]$$

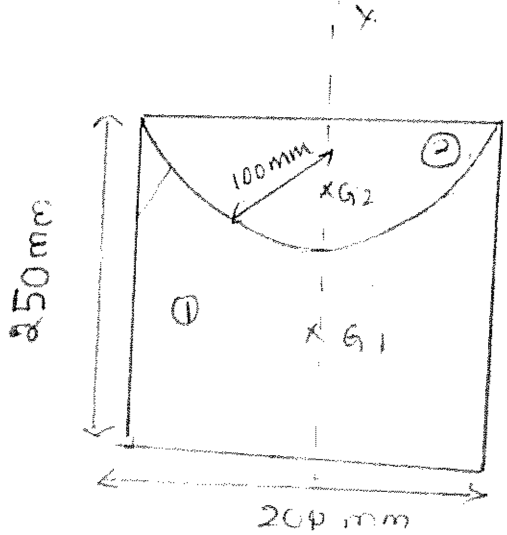
$$I_{yy} = 1.85 \times 10^6 \text{ mm}^4$$

$$I_{zz} = I_{xx} + I_{yy}$$

$$= 1.8 \times 10^6 + 1.85 \times 10^6$$

$$= 3.67 \times 10^6 \text{ mm}^4$$

10] For the area shaded in the figure find radius of gyration about centroidal horizontal axis. [K_{yy}]



$a_1 = 200 \times 250$ $\qquad y_1 = 125\,mm$

$a = 50{,}000\,mm^2$

$a_2 = -\frac{\pi \times 100^2}{2}$ $\qquad y_2 = 250 - \frac{4 \times 100}{3\pi}$

$a_2 = -15{,}700\,mm^2$ $\qquad y_2 = 207.56\,mm$

$\Sigma a = 34{,}300\,mm^2$

$\cancel{I_{xx}} = \quad \bar{y} = \frac{\Sigma a.y}{\Sigma a} = \frac{(50{,}000 \times 125) + (-15700 \times 207.56)}{34300}$

$\bar{y} = 87.21\,mm$

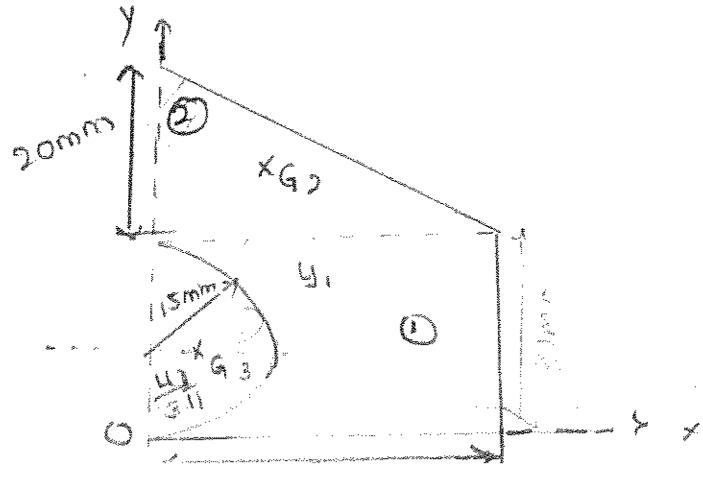
$I_{yy} = \left[\frac{200 \times 250^3}{12} + 200 \times 250 \times (125 - 87.2)^2\right] - \left[0.11 \times 100^4 + \frac{\pi \times 100^2}{2}(207.56 - 87.2)^2\right]$

$I_{yy} = 9.33 \times 10^7\,mm^4$

$k_{yy} = \sqrt{\frac{I_{yy}}{A}} = \sqrt{\frac{9.33 \times 10^7}{34{,}300}}$

$k_{yy} = 52.16\,mm$

10] For the area shaded in figure, find radius of gyra-tion about centroidal vertical axis.



$a_1 = 25 \times 30$ $x_1 = 12.5 mm$ y_1 ...

$a_1 = 750 mm^2$

$a_2 = \frac{1}{2} \times 25 \times 20$ $x_2 = \cancel{30} + \frac{1}{3} \times 20$ $y_2 = \frac{1}{3} \times 20 + 30$

$a_2 = 250 mm^2$ $x_2 = \cancel{36}.6 mm$ $= 36.6 mm$

$8.33 mm$

$a_3 = -\frac{\pi \times 15^2}{2} = -353.25 mm^2$ $X_3 = \frac{4 \times 15}{3 \times 3.14} = 6.\cancel{2}8 mm$

$y_3 = 15 mm$.

b) ~~$\Sigma a = 1353.25$~~

$\Sigma a = 646.75 mm^2$.

$\bar{x} = \frac{\Sigma a . x}{\Sigma a} = \frac{750 \times 12.5 + 250 \times \cancel{3}6.6 - 353.25 \times 6.\cancel{22}}{646.75}$

$\bar{x} = 13.61 mm$

a

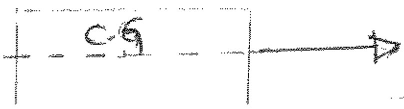
Unit - V

KINETICS

D. Alembert's principle of dynamic equilibrium :-

It states that "for any body, the algebraic sum of externally applied forces and the forces resisting the motion (or Inertia force) in any direction is zero".

F_I = Interia force
$F_I = ma$



F = external force

$F_I = \frac{W}{g} \cdot a$

$F - F_I = 0$

$F - ma = 0$, Here $W = mg$ $\therefore m = \frac{W}{g}$

$\therefore \boxed{F - \left(\frac{W}{g}\right) a = 0}$

The inertia force exists as a result of dynamic condition. The state of equilibrium thus created is calle-d "Dynamic equilibrum".

The equations of static equilibrium may be used to the conditions of dynamic equilibrium. These equation are $\Sigma H = 0$, $\Sigma V = 0$ and $\Sigma M = 0$.

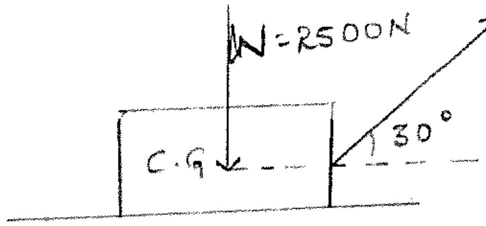
The equations of motion may be used to the problem on D. Alembert's principle of dynamic equilibrium.

They are (i) $v = u + at$

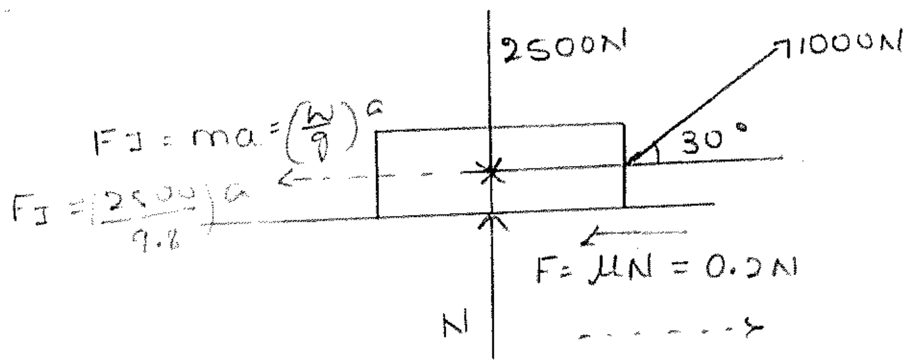
(ii) $s = ut + \frac{1}{2} a t^2$

(iii) $v^2 = u^2 + 2as$.

① A block weighing 2500N rests on a level horizontal plane which has a co-efficient of friction of 0.2. This block is pulled by a force of 1000N. which is acting at an angle of 30° to the horizontal. Find the velocity of the block after it moves 30m starting from the rest.



FBD



$$\Sigma V = 0$$

$$N - 2500 + 1000\sin 30° = 0$$

$$N = 2000\,N$$

$$\Sigma H = 0$$

$$1000\cos 30° - 0.2(2000) - \left(\frac{2500}{9.8}\right)a = 0$$

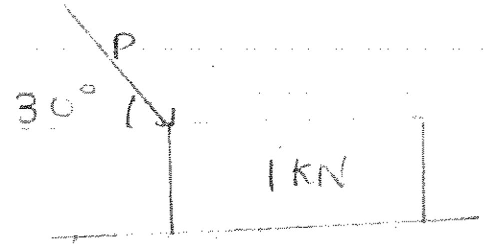
$$a = 1.83\ m/s^2$$

Using $v^2 = u^2 + 2as$

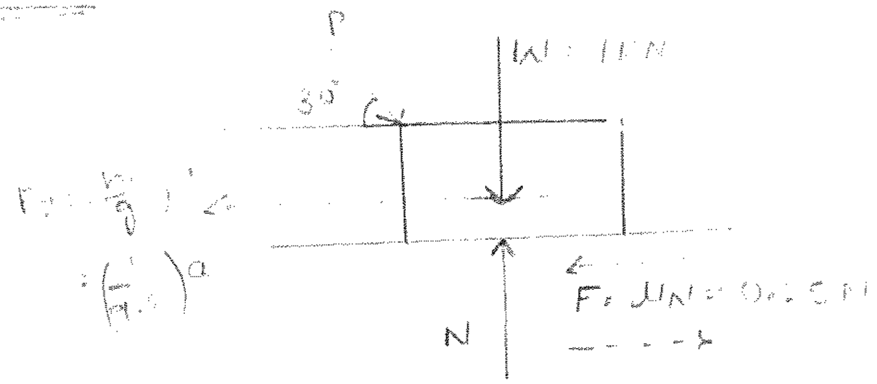
$$v^2 = 0 + 2(1.83)(30)$$

$$v = 10.48\ m/s$$

2] A block weighing 1KN rests on a horizontal plane as shown in the figure. Find the magnitude of the force P required to give the block an acceleration of 3 m/s² to the right. The co-efficient of friction between the block and the plane is 0.25.



FBD



$\Sigma V = 0$

$-1 - P\sin 30° + N = 0$

$N - P\sin 30 = 1 \quad \text{—Eq}(1)$

$\Sigma H = 0$

$-P\cos 30° - 0.25N - \left(\frac{1}{9.8}\right)3 = 0$

$-0.25N + P\cos 30° = 0.3058 \quad \text{—Eq}(2)$

By solving (1) & (2)

$P = 0.75 \text{ KN}$

$N = 1.375 \text{ KN}$

3) A motorist travelling at a speed of 70 Kmph, suddenly applies brakes and halts after skidding 50 m.
Determine (i) The time required to stop the car
July 2023 (ii) The co-efficient of friction between the tyre and the road.

$u = 70$ Kmph

$v = 0$

$s = 50$ m

$u = 70 \times \left(\frac{5}{18}\right) = 19.44$ m/s

(i) = $t = ?$

(ii) $\mu = ?$

using $v^2 = u^2 + 2as$

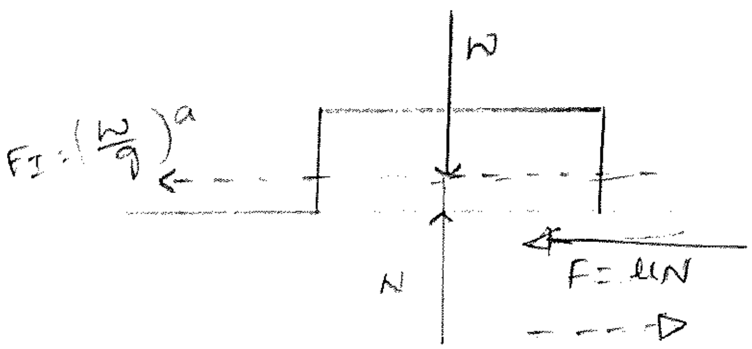
$$0 = (19.44)^2 + 2(a)(50)$$

$$a = -3.78 \ m/s^2$$ (Retardation)

$$v = u + at$$

$$0 = 19.44 + (-3.78)t$$

$$t = 5.14 s$$



$$\Sigma V = 0$$

$$N - W = 0$$

$$N = W$$

$$\Sigma H = 0$$

$$-\mu N - \left(\frac{W}{g}\right)a = 0$$

$$-\mu W - \left(\frac{W}{g}\right)(-3.78) = 0$$

$$-\mu W + \frac{3.78 W}{g} = 0$$

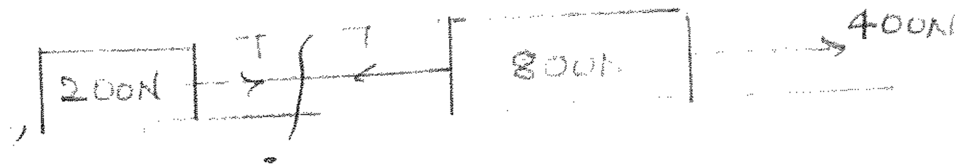
$$\frac{-g\mu W + 3.78 W}{g} = 0$$

$$W(-\mu g + 3.78) = 0$$

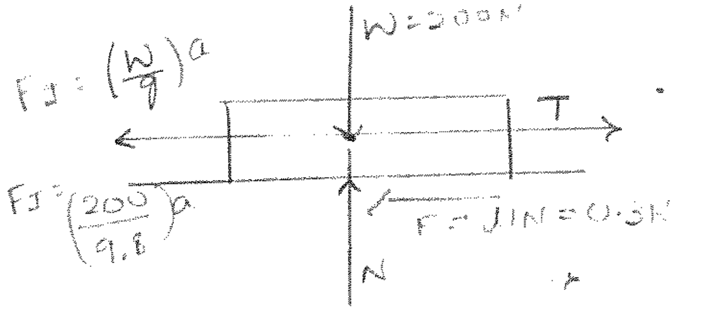
$$-\mu \times 9.8 + 3.78 = 0$$

$$\mu = \frac{3.78}{9.8} \quad \mu = 0.385$$

4] Two weights 800N and 200N are connected by a thread and they move along a rough horizontal plane under the action of a force of 400N applied to the 800N weight as shown in the figure. The co-efficient of friction between the sliding surface of the weights and the plane is 0.3. Using D. Alembert's principle, determine the acceleration of the weight and tension in the thread.



FBD of weight 200N



$\Sigma V = 0$

$N - 200 = 0$

$N = \underline{200N}$

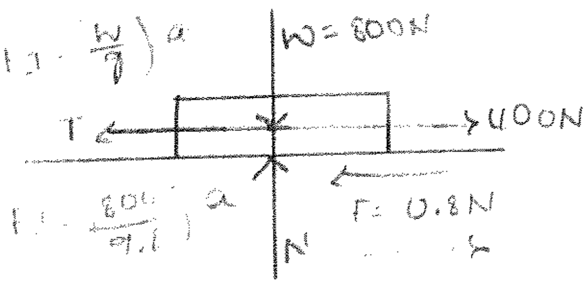
$\Sigma H = 0$

$T - 0.3N - \left(\frac{200}{9.81}\right)a = 0$

$T - (0.3)(200) - \left(\frac{200}{9.81}\right)a = 0$

$T - 20.387a = 60 \rightarrow$ eq (1)

FBD of weight 800N



$\Sigma V = 0$

$N - 800 = 0$

$N = \underline{800N}$

$\Sigma H = 0$

$-T - \left(\frac{800}{9.81}\right)a - 0.3(800) + 400 = 0$

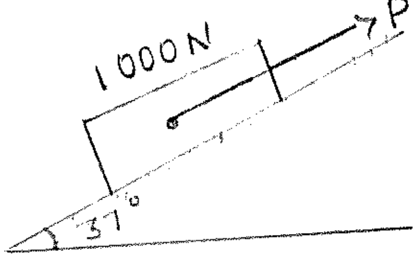
$-T - 81.55a = -160 \rightarrow$ eq (2)

By solving (1) and (2)

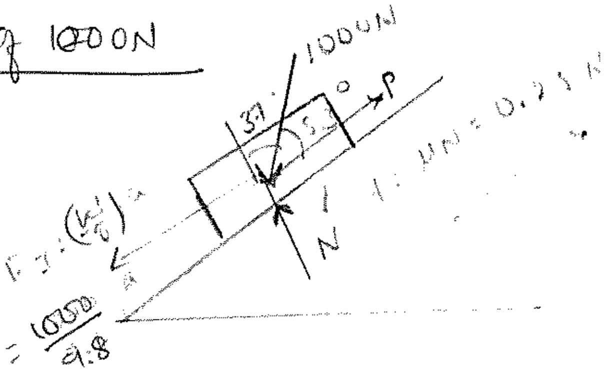
$T = 79.99 = \underline{80\ N}$

$a = 0.98\ m/s^2$

5] A body weighing 1000 N is placed on a rough inclined plane. The co-efficient of friction between the body and the inclined plane is 0.25. A force of magnitude P is applied to the body parallel to the plane as shown in the figure. Determine the value of P required to bring the body from rest to a velocity of 6 m/s in 3 seconds.



FBD of 1000 N



$\Sigma F_{\perp r} = 0$

$N - 1000 \sin 53° = 0$

$N = \underline{798.63\ N}$

$\Sigma F_{\parallel} = 0$

$P - 0.25N - \frac{1000}{9.8}a - 1000 \cos 53° = 0$

$P = 801.47 + 101.94a$

To find 'a', using $v = u + at$

$6 = 0 + a(3)$

$a = \underline{2\ m/s^2}$

$P = 801.47 + 101.94(2)$

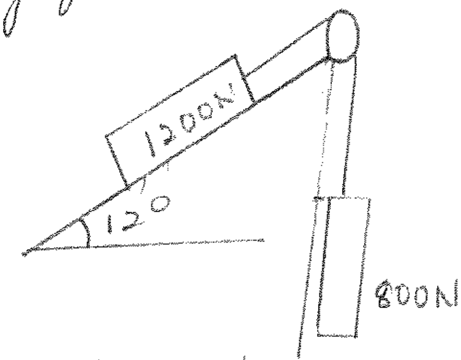
$P = 1005.35\ N$

6] A body weighing 1200N rests on a rough plane inclined at 12° to the horizontal. It is pulled up the plane and by means of a rope running parallel to the plane and passing over a frictionless pulley at the top of the plane as shown in the figure. The portion of the rope beyond the pulley hangs vertically down and carries a weight of 800N at its end. If $\mu = 0.2$ find

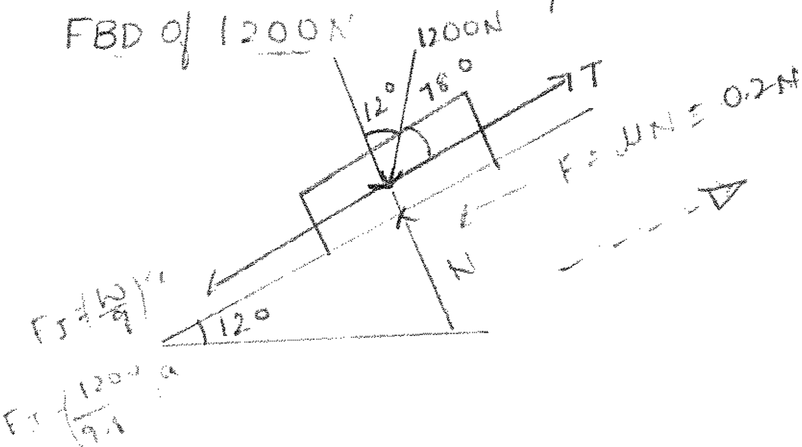
a] Tension in the rope.

b] Acceleration with which the body moves up the plane

c] The distance moved by the body moves in 3 seconds after starting from rest.



FBD of 1200N



$\Sigma F_{\perp r} = 0$

$N - 1200 \sin 78° = 0$

$N = 1173.78\,N$

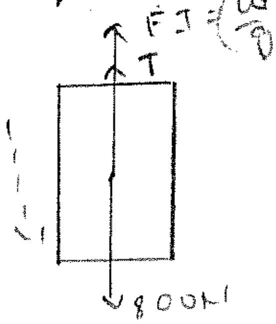
$\Sigma F_{\parallel} = 0$

$T - 0.2N - 1200 \cos 78° - \left(\frac{1200}{9.81}\right) a = 0$

$T - (0.2)1173.78 = 484.25 \rightarrow ①$

$T - 122.32a = 484.25 \rightarrow ①$

FBD of 800N



$$\Sigma V = 0$$

$$T + \left(\frac{800}{9.81}\right)a - 800 = 0$$

$$T + 81.55a = 800 \quad \text{--- (2)}$$

solving (1) and (2)

$$T = 67369\ N$$

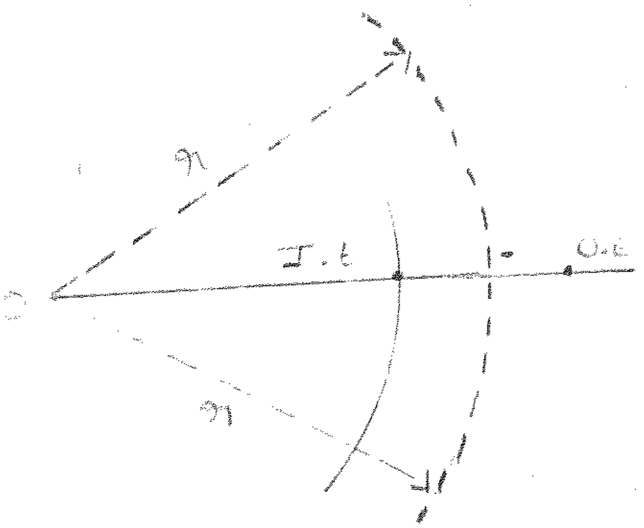
$$a = 1.55\ m/s^2$$

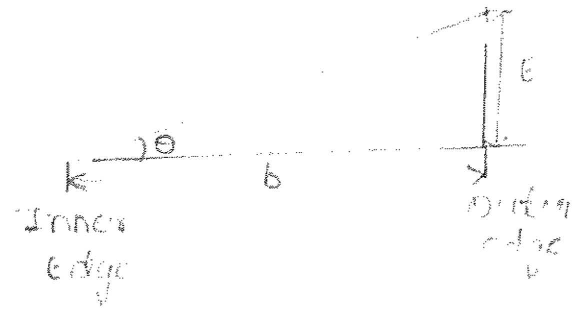
using $s = ut + \frac{1}{2}at^2$

$$s = 0 + \frac{1}{2}(1.55)(3)^2$$

$$s = 6.975\ m$$

MOTION ALONG A CIRCULAR CURVE





When a vehicle takes a curved path with some velocity on outward centrifugal force acts on it, which may cause the vehicle to skid or overturn.

Banking (E) : Raising of outer edge of pavement over an inner edge on highway curves.

Superelevation (E) : Raising of outer rail over an inner rail on railway curves.

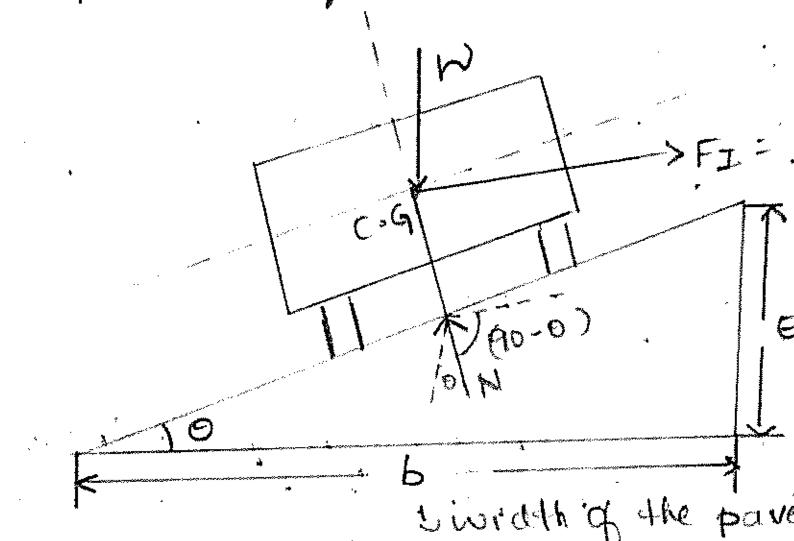
Banking and superelevation are provided to avoid undesirable conditions on highway curves and railway curves, such as skidding and overturn-ing.

This banking or superelevation that should be given depends on speed of the vehicle (V) and the radius of curvature (r).

<u>By providing the banking or superelevation the following are achieved</u> :-

(i) Skidding and overturning are avoided.
(ii) Higher speed may be premitted on a curved path.
(iii) Lateral pressure may be eliminated or reduced, giving more comfort to the passengers.
(iv) Wear and tear of wheels is reduced.

Expression for super elevation (E) :-



$F_I = m\frac{(V^2)}{r}$

$= \frac{W}{g}\frac{V^2}{r}$

θ = Angle of Banking (or) Superelevation

F_I = Centrifugal Intertia force $= \frac{mv^2}{r} = \frac{Wv^2}{gr}$

[F_I acts radially outwards and is horizontal)

$\Sigma H = 0,$

$$F_I - N\cos(90 - \theta) = 0$$

$$\frac{Wv^2}{gr} - N\sin\theta = 0$$

$$\boxed{N\sin\theta = \frac{Wv^2}{gr}} \rightarrow (1)$$

$\Sigma V = 0$

$$N\sin(90 - \theta) - W = 0$$

$$\boxed{N\cos\theta = W} \rightarrow (2)$$

[Dividing (1) by (2) gives]

$$\frac{(1)}{(2)} = \frac{N\sin\theta}{N\cos\theta} = \frac{Wv^2/gr}{W}$$

$$\therefore \boxed{\tan\theta = \frac{V^2}{gr}} \rightarrow (3)$$

But $\tan\theta = \frac{E}{b}$ $\therefore \frac{E}{b} = \frac{V^2}{gr}$ $\boxed{E = \frac{V^2 b}{gr}}$

Design Speed or Normal Speed :-

The speed on a banked curved path for which no lateral pressure, develops is called the "design speed" or Normal speed" on that curve.

$$\tan\theta = \frac{V^2}{gr} \quad \therefore V = \sqrt{gr\tan\theta}$$

[If the vehicle moves with design speed there will be equal pressure on inner and outer wheels and hence passengers will not experience discomfort. For design speed there will not be any lateral frictional force and wheel reaction are normal to road]

Maximum Speed :-

The maximum speed is the speed at which the vehicle can travel round a curve without skidding outwards.

$$\tan(\theta+\phi) = \frac{V^2_{max}}{gr}$$

μ = Coeff of friction

But $\mu = \tan\phi \therefore \phi = \tan^{-1}\mu$.

Here ϕ = Angle of friction

$$\therefore V_{max} = \sqrt{gr\tan(\theta+\phi)}$$

$$= \sqrt{gr\left(\frac{\tan\theta + \tan\phi}{1-\tan\theta.\tan\phi}\right)}$$

$$V_{max} = \sqrt{gr\left(\frac{\mu+\tan\theta}{1-\mu\tan\theta}\right)}$$

Minimum Speed :-

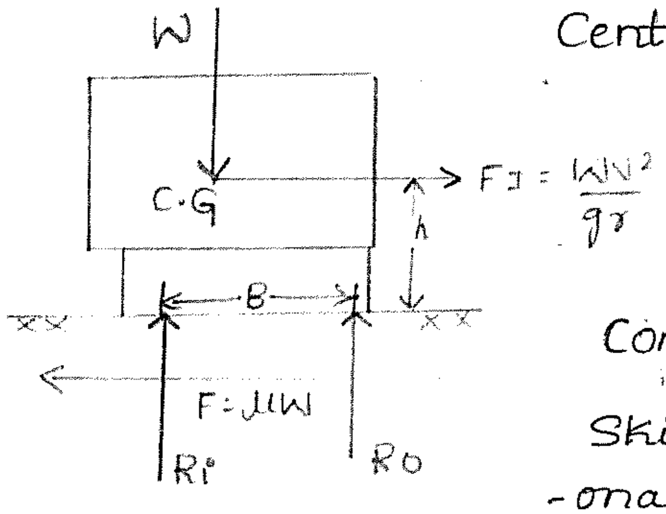
The minimum speed is the speed at which the vehicle can travel round a curve without skidding inwards.

$$\tan(\theta-\phi) = \frac{V^2_{min}}{gr}$$

$$V_{min} = \sqrt{gr\tan(\theta-\phi)}$$

Motion on a level road [or level circular path]

[or Flat curve]



Centrifugal Inertia force (F_I)

$= \frac{mv^2}{r}$, But, $m = \frac{W}{g}$

$\Sigma V = 0$

$R_i + R_o = W$

$F = \mu(R_i + R_o)$

$F = \mu W$

Condition for Skidding :-

Skidding takes place when frictional force reaches its limiting value i.e $F = \mu W$

$\Sigma H = 0$

$$\frac{WV^2}{gr} - \mu W = 0$$

$$\frac{\cancel{W}v^2}{gr} = \mu \cancel{W}$$

Maximum speed (or limiting speed) to avoid skidding $\boxed{V = \sqrt{\mu g r}}$

Where μ = coeff of friction between the wheels of the vehicle and the road.

$[\mu = \tan\phi]$

This is the maximum speed (velocity) to avoid skidding away of a vehicle on level circular path.

[This is the maximum speed at which to vehicle can travel round the level circular path without skidding outwards]

Condition for overturning :-

Overturning may take place against outer wheel, when the vehicle is about to overturn, $R_i = 0$

∴ Taking moments about the outer wheel

Σ Moment about the outer wheel = 0

$\Sigma M_0 = 0$

$$\therefore -W\frac{B}{2} + \frac{WV^2}{gr} \times h = 0$$

$$\therefore \frac{\cancel{W}V^2}{gr} \times h = \cancel{W}\frac{B}{2}$$

Maximum speed [or limiting speed] to avoid overturning $\boxed{V = \sqrt{\frac{grB}{2h}}}$

This is the maximum velocity to avoid over turning of a vehicle on a level circular path.

Problems

01] A highway curve has a width of 6m and a radius of 130m. Find a] The banking required for normal speed of 50 kmph.

b] The maximum speed at which the vehicle may travel without skidding on the curve.

If $\mu = 0.25$ between the tyre and the pavement.

02] Calculate the angle of superelevation and super elevation of the rail on a curved track for a locomotive running at 60 kmph, gauge and radius of curvature being 1.68m and 800m respectively. Find the lateral thrust [or lateral force (F)] exerted on the outer rail, if the speed of the locomotive is changed to 80 kmph weight of the locomotive is 1000 kN.

03] A car weighing 20 KN, goes round a curve of 60m radius banked at 30°. Find the frictional force acting on the tyres and normal reaction on outer and inner wheels, when the car is travelling at 96 Kmph. The Coefficient of friction between the tyre and the road is 0.6. Take width of the wheel base, $B = 1.6\,m$ and height of C.G of the vehicle above the road level $h = 0.8\,m$.

04] A car weighing 15 KN goes round a flat curve (level road (or) level circular path) of 50m radius. The distance between inner and outer wheel is 1.5 m and the C.G is 0.75m above the road level. What is the limiting speed of the car on the curve? Determine the normal reactions developed at the inner and outer wheels, if the car negotiates the curve with a speed of 40 Kmph. Take the Coefficient of friction, $\mu = 0.4$.

05] Find at what speed a vehicle can move round a curve of 40m radius without slide slip (i.e skidding)

(i) on a level road (Flat curve)

(ii) on a road banked to an inclination of 1 in 10.

At what speed can the vehicle travel on banked road without any lateral frictional force?

Take $\mu = 0.4$.

01] Given :-

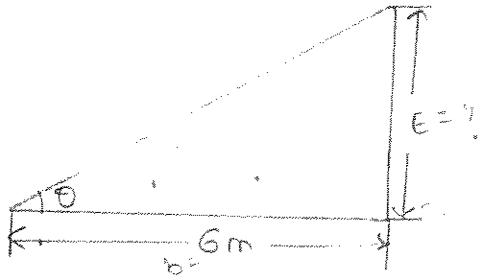
$b = 6m$

$r = 130m$

a] To find banking if $V = 50 kmph$

$$= 50 \times \frac{5}{18}$$

$$= 13.88 \ m/s.$$



$$\tan\theta = \frac{v^2}{gr}$$

$$\frac{E}{b} = \frac{v^2}{gr}$$

$$E = \frac{v^2 b}{gr}$$

$$E = \frac{(13.88)^2 \times 6}{9.81 \times 130}$$

$$E = \underline{0.906 \ m}$$

b] $V_{max} = ?$ $\quad \mu = 0.25$

$$V_{max} = \sqrt{gr \tan(\theta + \phi)}$$

$$\tan\theta = \frac{E}{b} = \frac{0.906}{6}$$

$$\tan\theta = 0.151$$

$$\boxed{\theta = 8.59^\circ}$$

$$\mu \tan\phi \ \therefore \ \phi = \tan^{-1}\mu$$

$$= \tan^{-1}(0.25)$$

$$\phi = 14.03^\circ$$

$$V_{max} = \sqrt{9.81 \times 130 \tan(8.59 + 14.03^\circ)}$$

$$V_{max} = 23.05 \ m/s$$

$$V_{max} = 23.05 \times \frac{18}{5}$$

$$V_{max} = \underline{83.02\ kmph}$$

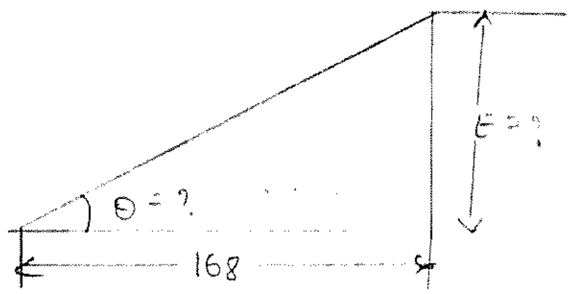
Q2] $b = 1.68m$

$r = 800m$

$V = 60\ kmph$

$V = 60 \times \frac{5}{18}$

$V = 16.66\ m/s.$



$$\tan\theta = \frac{V^2}{gr}$$

$$= \frac{(16.66)^2}{9.81 \times 800}$$

$$\theta = \underline{2.004^\circ}$$

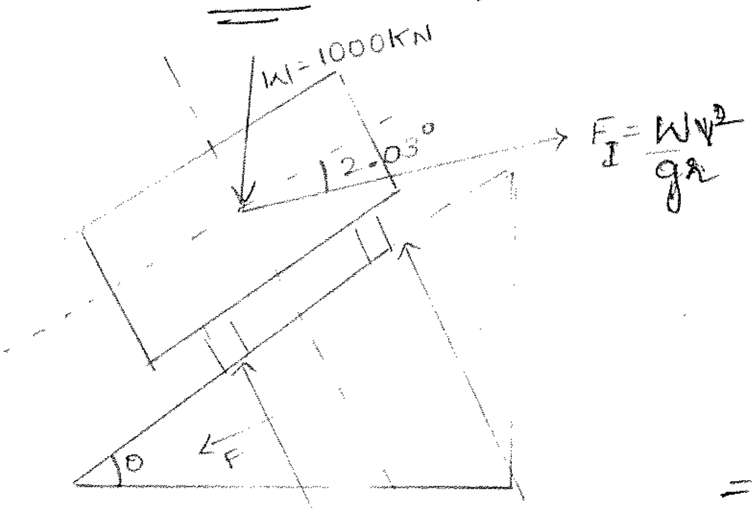
$$\frac{E}{b} = \frac{V^2}{gr}$$

$$E = \frac{V^2 b}{gr}$$

$$E = \frac{(16.67)^2 \times 1.68}{9.81 \times 800}$$

$$E = \underline{0.059\ m}$$

b]



$$\frac{WV^2}{gr}$$

$$= \frac{1000 \times (16.67)^2}{9.81 \times 1.68}$$

$$= \underline{62.91\ KN}$$

~~03~~ $\Sigma F_{\mu} = 0$

$-F + 62.91 \cos 2.03 - 1000 \cos 87.97 = 0$

Lateralforce (or frictional force)

$\underline{\underline{F = 27.45 KN}}$

3] $W = 20KN$

$r = 60m$

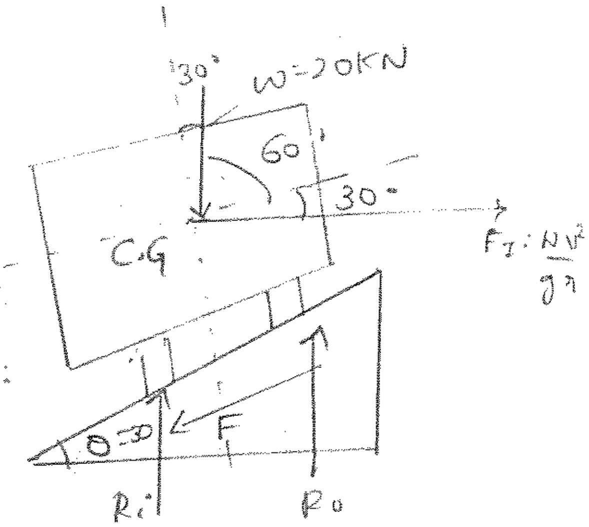
$V = 96 Kmph$

$= \frac{96 \times 5}{18}$

$V = 26.66 m/s$

$B = 1.6m$

$h = 0.8m$



$F_I = \frac{WV^2}{gr}$

$F_I = \frac{20 \times (26.66)^2}{9.81 \times 60}$

$\underline{\underline{F_I = 24.17 KN}}$

$\Sigma M_F = 0$

$-F - 24.17 \cos 30° - 20 \cos 60° = 0$

lateral force (or frictional force)

$\boxed{F = 10.93 KN}$

To find Ri and Ro

$\Sigma F_{\perp} = 0$

$R_i + R_o = 20 \sin 60 + 24.17 \sin 30$

$R_i + R_o = 24.41$

$\Sigma m_i = 0$

$-R_o \times 1.6 = 20\cos 60^\circ \times 0.8 + 20\sin 60 \times 0.6 + 24.17$

$R_o = 20.17\ kN$

$R_i = 29.41 - 20.17$

$R_i = 9.24\ kN$

04] a] To find limiting speed of the car on flat curve. limiting speed to avoid skiding (maximum speed)

$V = \sqrt{\mu g r}$

$= \sqrt{0.4 \times 9.81 \times 50}$

$V = 14\ m/s = 50.4\ Kmph$

$\frac{NV^2}{gr}$

b] Limiting speed or max speed to avoid overturning

$V = \sqrt{\frac{grb}{2h}} = \sqrt{\frac{9.81 \times 50 \times 1.5}{2 \times 0.75}}$

$= 22.15\ m/s$

$= 22.15 \times \frac{18}{5}$

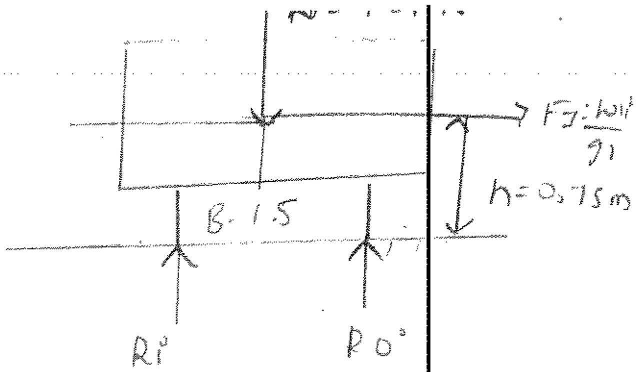
$V = 79.74\ Kmph$

limiting speed (or Max speed of the car) $V = 50.4\ Km$

b] $R_i = ?$ $R_o = ?$ if $V = 40\ Kmph$

$= 40 \times \frac{5}{18}$

$= 11.11\ m/s$



$$F_I = \frac{WV^2}{gr}$$

$$F_I = \frac{15 \times (11.11)^2}{9.81 \times 50}$$

$$F_I = \underline{\underline{3.775 KN}}$$

$$\Sigma V = 0$$

$$R_i + R_o = 15$$

$$\Sigma M_o = 0$$

$$(R_i \times 1.5) + 3.775 \times 0.75 - 15 \times 0.75 = 0$$

$$R_i = \underline{\underline{5.612 KN}}$$

$$R_o = 15 - 5.612$$

$$R_o = \underline{\underline{9.388}} \ KN$$

5] (i) on a level road [flat curve)

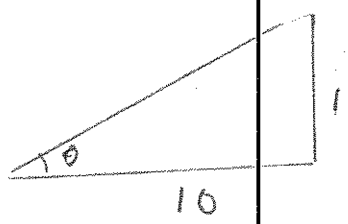
$$V = \sqrt{\mu g r}$$

$$V = \sqrt{0.4 \times 9.81 \times 40}$$

$$V = \underline{\underline{12.528}} \ m/s$$

$$V = 12.528 \times \frac{18}{5}$$

$$V = \underline{\underline{45.1008 \ kmph}}$$



$$V_{max} = \sqrt{gr \tan(\theta + \phi)}$$

$$\tan\theta = \frac{1}{10}$$

$$\theta = \tan^{-1}(1/10)$$

$$\theta = \underline{\underline{5.71^\circ}}$$

$$\mu \tan\phi = 0$$

$$\phi = \tan^{-1}(\mu)$$

$$\phi = \tan^{-1}(0.4)$$

$$\phi = \underline{21.80^\circ}$$

$$V_{max} = \sqrt{9.81 \times 40 \times \tan(5.71 + 21.80)}$$

$$V_{max} = 14.29\ m/s$$

$$V_{max} = 14.29 \times \frac{18}{5}$$

$$V_{max} = \underline{\underline{51.46\ Kmph}}$$

(ii) $\tan\theta = \frac{V^2}{gr}$

$\cancel{\tan\theta =}$ $V = \sqrt{gr\tan\theta}$

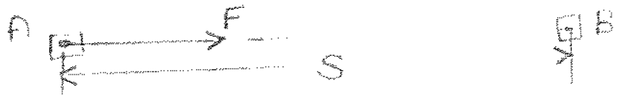
$$V = \sqrt{9.81 \times 40 \times \tan(5.71^\circ)}$$

$$\boxed{V = 22.536\ \underline{Kmph}}$$

WORK AND ENERGY

WORK :-

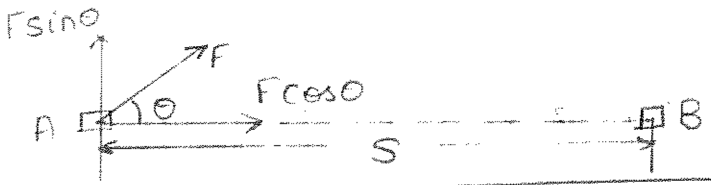
The work done by a force on a moving body is defined as the product of the force and the distance moved in the direction of the force.



$\boxed{\text{Work done} = F \times S}$

N-m [or Joule (J)]

kN-m [or kilo joule (kJ)]



$\boxed{\text{Work done} = F\cos\theta \times S}$

Energy :-

The energy is defined as the capacity to do the work, energy is also measured by Joules (J) or kilo joules (kJ).

a] Potential Energy :-

It is the capacity to do the work due to the position of the body.

$\therefore$ Potential Energy $= mgh = W.h$

b] Kinetic Energy :-

It is the capacity to do the work due to the motion of the body.

$\therefore$ Kinetic energy $= \frac{1}{2} m v^2 = \frac{1}{2}\left(\frac{W}{g}\right) v^2 = \frac{Wv^2}{2g}$

Work - Energy principle :-

It states that "Work done on a particle is equal to its change in kinetic energy.".

Work done = change in kinetic energy

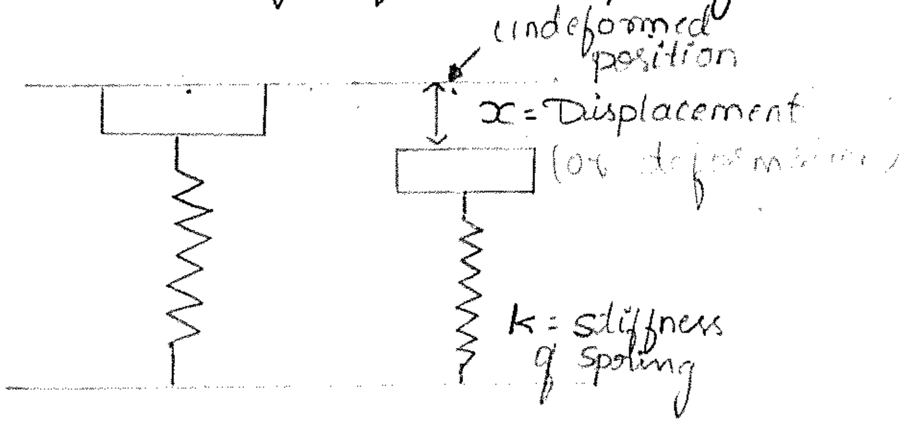
= Final KE - Initial K.E

$$= \frac{Wv^2}{2g} - \frac{Wu^2}{2g}$$

$$\therefore \boxed{\text{Work done} = \frac{W}{2g}(v^2-u^2)}$$

This equations called work - Energy equation.

Work done by a force on Spring :-



The work done by a force on spring of stiffness 'k' to a displacement 'x', from undeformed position is,

$$u = -\frac{1}{2} k . x^2$$

Here 'k' is called spring constant [or stifness of spring or modulus of spring] and is defined as a force required for unit deformative of the spring. Hence the unit of spring constant is N/mm, N/m or kN/m.

Virtual work [or Imaginary work] :-

The work done by a force on a body due to small virtual [or Imaginary] displacement of the body is known as virtual work.

∴ Virtual work = Force x virtual displacement
[or imaginary displacem-ent]

Principle of virtual work :-

It states that "If a system of forces acting on a body or system of bodies be in equilibrium and if the system is imagined to undergo a small displacem-ent (i.e virtual displacement), then the algebraic sum of the virtual work done by the forces of the system is zero."

Power :-

Power is defined as time ratio of doing work.

unit of Power is watt.

It is defined as one Joule of work done in one second

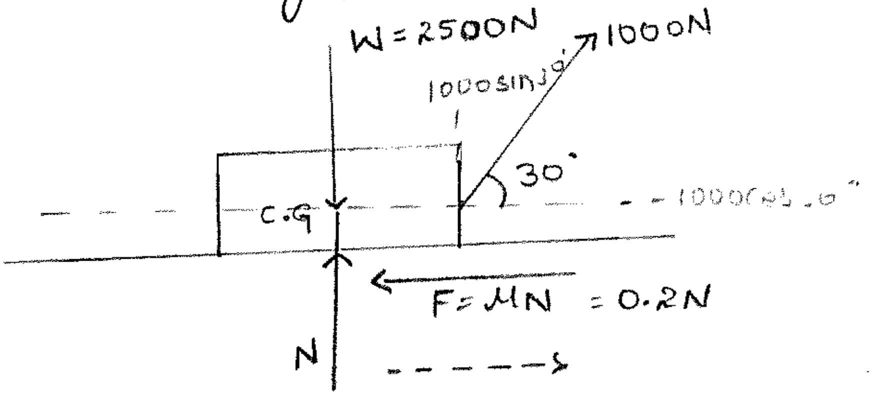
1 Kilowatt = 1000 watts.

Problems :-

01] A block weighing 2500N rests on a level horizontal plane for which the coefficient of friction is 0.2. This block is pulled by a force of 1000 N, acting at an angle of 30° to the horizontal. Find the velocity of the block after it moves 30m starting from rest. Use work-energy method

S = 30m

u = 0



$\Sigma V = 0$

$N - 2500 + 1000 \sin 30° = 0$

$\underline{N = 2000 \text{ Nt}}$

Work done $= (1000 \cos 30) - (0.2 \times 2000) \times 30$

$= \underline{13{,}980.76 \text{ N-m}}$

Change in K.E $= \frac{W V^2}{2g} - \frac{W u^2}{2g}$

$= \frac{W}{2g}(V^2 - u^2)$

$= \frac{2500}{2 \times 9.8}(V^2)$

$= \underline{127.42 V^2}$

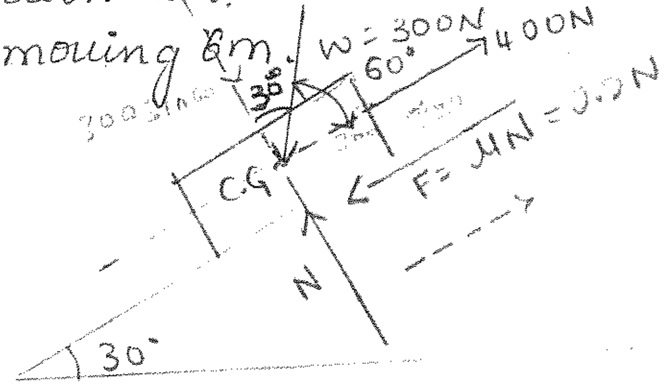
Applying work-energy principle = work done = change in KE

$= 13{,}980.76 = 127.42 V^2$

$\boxed{V = 10.47 \text{ m/s}}$

Q2] A body weighing 300N is pushed up a 30° by plane by a 400N force acting parallel to the plane. If the initi-al velocity of the body is 1.5 m/s and co-efficient of Kinetic friction is, $\mu = 0.2$, what velocity will the body have after moving 6m.

$S = 6m$
$u = 1.5 m/s$
$v = ?$



$\Sigma F_y = 0$

$N - 300 \sin 60° = 0$

$N = \underline{\underline{259.81 \text{ Nt}}}$

Workdone $= (400 - 300 \cos 60° - 0.2 \times 259.81) \times 6$

$= \underline{\underline{1188.22 \text{ N-m}}}$

Change in K.E $= \dfrac{Wv^2}{2g} - \dfrac{Wu^2}{2g}$

$= \dfrac{W}{2g}(v^2 - u^2)$

$= \dfrac{300}{2 \times 9.81}(v^2 - (1.5)^2)$

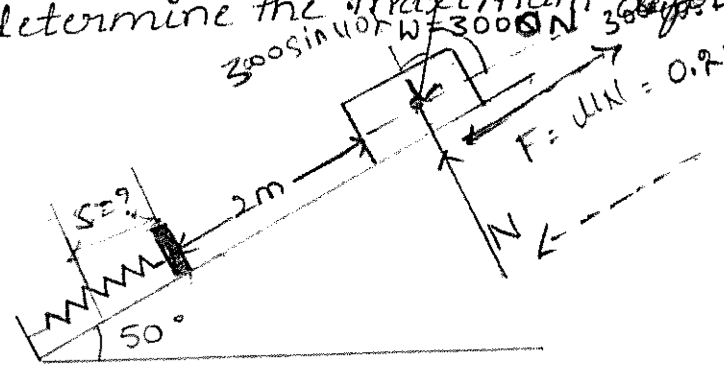
$= 15.29 (v^2 - 1.5^2)$

Applying work-energy eq^n =' workdone = Change in KE

$= 1188.22 = 15.29 (v^2 - 1.5^2)$

$\boxed{v = 8.94 \text{ m/s}}$

Q3] A 3000N block starting from ... as shown in the figure. Slides down a 50° inclined plane. After moving 2m it strikes a spring whose modulus is 20 N/mm. If the co-efficient of friction b/w the block and the inclined plane is 0.2, determine the maximum deformation of the spring.



$K = 20 N/mm$

$u = 0$

$2m = 2000mm$

$\Sigma F_{\perp} = 0$

$N - 3000\sin 40° = 0$

$N = 1928.36 Nt$

Workdone by a force $= (3000\cos 40° - 0.2 \times 1928.36)(2000+S)$

$= 1912.46(2000+S)$

Workdone by a Spring $= -\frac{1}{2} Kx^2 = -\frac{1}{2} \times 20 \times S^2$

$= -10S^2$

Change in KE $= \frac{W}{2g}(v^2 - u^2)$

$= \frac{3000}{2 \times 9.81}(0 - 0)$

$= 0$

Applying Work-energy eq^n = Workdone by a force & spring − Change in KE

$3824920 + 1912.46S - 10S^2$

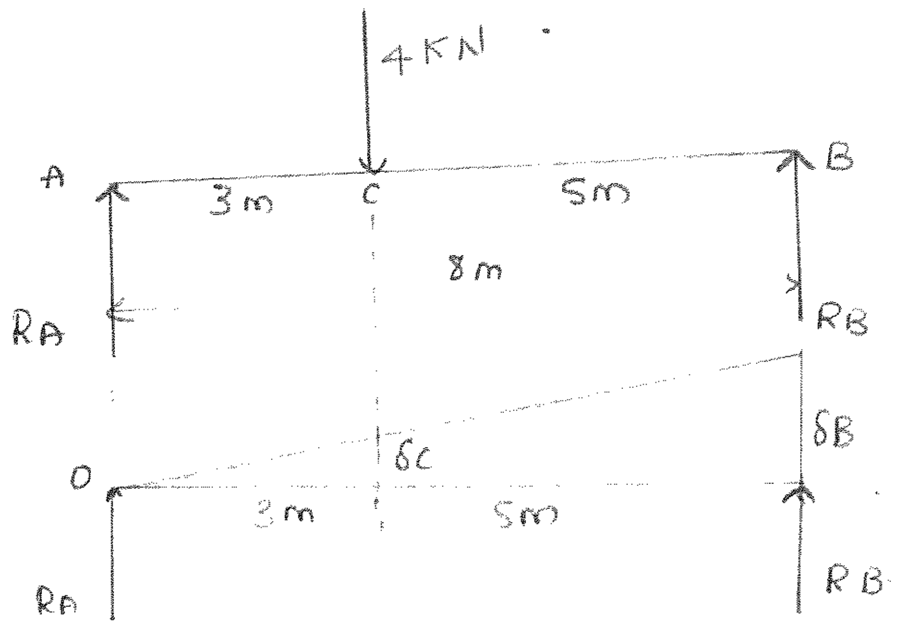
$1912.46(2000+S) - 10S^2 = 0$

$3824920 + 1912.46S - 10S^2 = 0$

$+10S^2 - 1912.46S - 3824920 = 0$

$= 721.43\ mm$

Q4) Using the principle of virtual work, determine the reactions of a simply supported beam, AB of span 8m. The beam carries a point load of 4 KN at a distance of 3m from the left supported A.



$$\frac{\delta C}{3} = \frac{\delta B}{5}$$

$$\delta C = \frac{\delta B \times 3}{5}$$

Alegraabic Sum of virtual work done by all the forces =

$$= (R_B \times \delta B) + (R_A \times 0) - (4 + \delta C)$$

$$= R_B \times \delta B - 4 \times \left(\frac{\delta B}{5}\right) 3$$

$$= R_B \times \delta B - 1.5\, \delta B.$$

From the principle of virtual work : The algebraic sum of the virtual work done should be zero.

$$R_B \times \delta B - 1.5 \delta B = 0$$

$$\delta B (R_B - 1.5) = 0$$

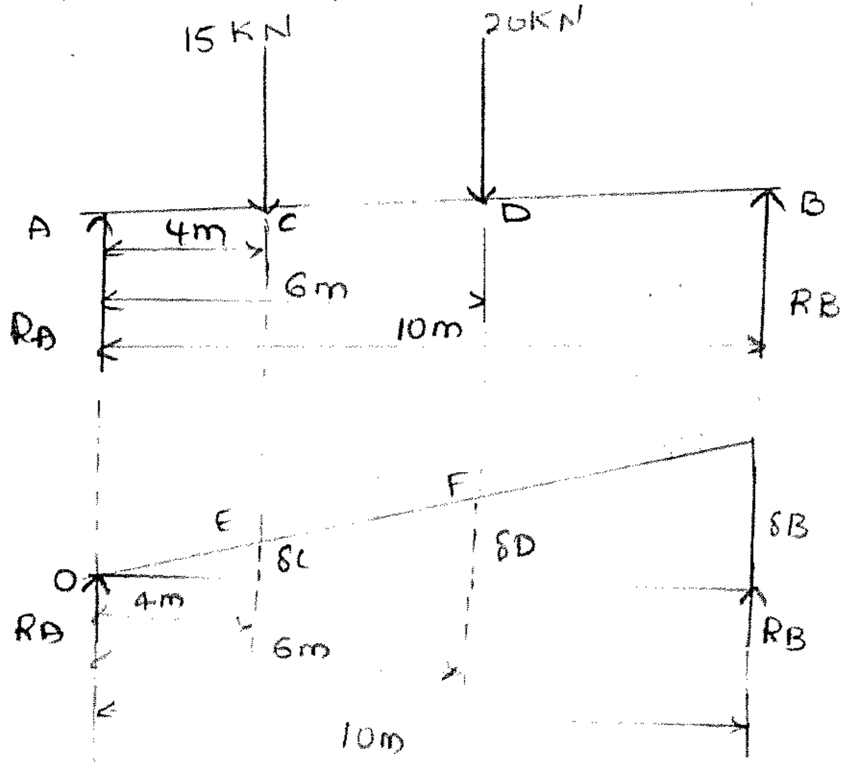
$$\boxed{R_B = 1.5 KN}$$

But $\Sigma V = 0$, $R_A + R_B = 4$

$R_A = 4 - 1.5$

$$\boxed{R_A = 2.5 KN}$$

05] A simply supported beam AB of span 10m carries two point loads of 15 KN and 20 KN at 4m and 6m from the left end A respectively. Determine the beam reactions by the principle of virtual work.



$$\frac{\delta C}{4} = \frac{\delta D}{6} = \frac{\delta B}{10}$$

$\cancel{\delta C = \delta D \times 4}$ $\quad \delta C = \frac{4}{10}\delta B$

$$\boxed{\delta C = 0.4\delta B}$$

$$\delta D = \frac{\delta B\, 6}{10}$$

$$\boxed{\delta B = 0.6\,\delta B}$$

Alegrabic sum of virtual workdone by all the force :

$$(RB \times \delta B) + (RA \times 0) - (15 + \delta C) - (20 \times \delta D)$$

$$RB \times \delta B - 15 \times 0.4\delta B - 20 \times 0.6\delta B$$

$\cancel{RB \times \delta B - 7.2\delta B}$, $\quad RB \times \delta B - 18\,\delta B$

$\cancel{RB \times \delta B - 6\delta B}$

$R_B \times \delta_B - 18\,\delta_B = 0$

$\delta_B (R_B - 18) = 0$

$\underline{\underline{R_B = 18\ KN}}$

But $\Sigma V = 0$

$R_A + R_B = 35$

$R_A = 35 - R_B$

$= 35 - 18$

$= \underline{\underline{17\ KN}}$

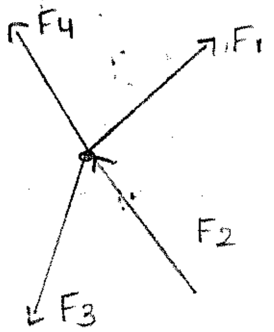
UNIT - 01

1. a (i) Explain briefly the classification of force system.

* Coplanar forces :-

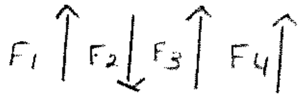
Forces are acting in the same plane.

* Coplanar Concurrent forces :-



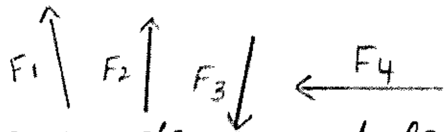
Forces are acting in the same plane and meeting at a point.

* Coplanar parallel force :-



Forces are acting in the same plane and are parall to each other.

* Coplanar non-current forces :-



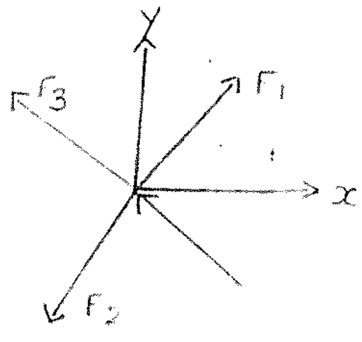
Forces are acting in the same plane and forces neither meet at a point nor parallel to each other.

Non-Coplanar forces :-

Forces are acting in different planes.

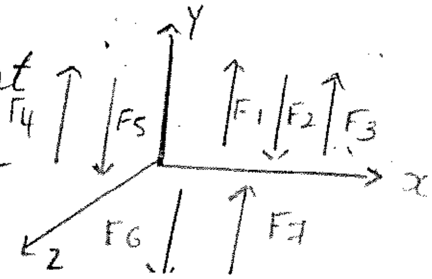
Non-Coplanar Concurrent forces :-

Forces are acting in different planes and meeting at a point.



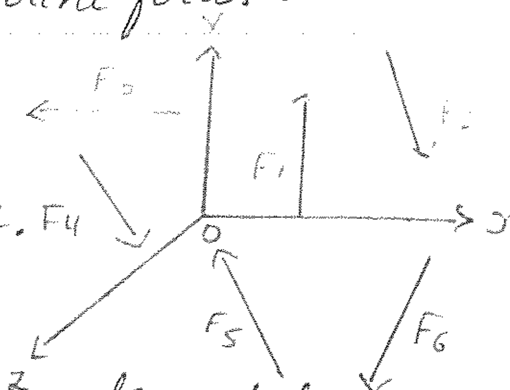
Non-Coplanar parallel forces :-

Forces are acting in a different planes and are parallel to each other.



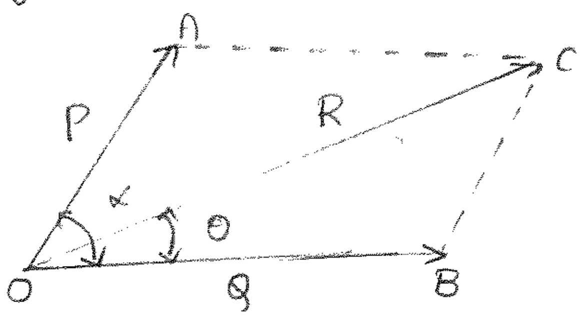
Non-coplanar and Non-Concurrent forces :-

Forces are acting in different planes and forces neither meet at a point nor parallel to each other.



(ii) State and Explain Parallelogram law of forces.

It states that "If two forces, which act at a point be represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from one of its angular points, their resultant (R) is represented by the diagonal of the parallelogram passing through that angular point in magnitude and direction.



$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$

$$\tan\theta = \frac{P\sin\alpha}{Q + P\cos\alpha}$$

1 b] A System of four forces acting at a point on a body is as shown in the figure. Determine the resultant.

$\tan\theta_1 = \frac{1}{2}$

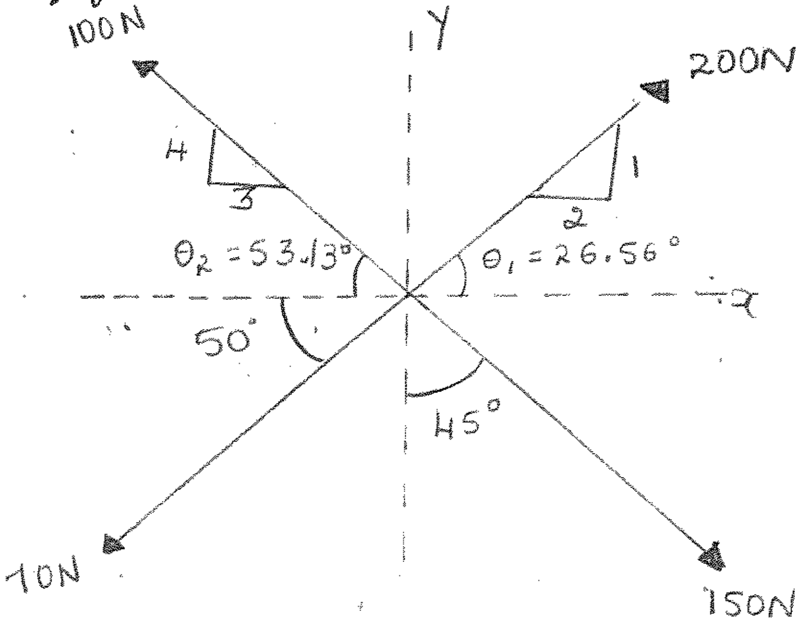
$\theta_1 = \tan^{-1}(1/2)$

$\boxed{\theta_1 = 26.56^\circ}$

$\tan\theta_2 = \frac{4}{3}$

$\theta_2 = \tan^{-1}(4/3)$

$\boxed{\theta_2 = 53.13^\circ}$



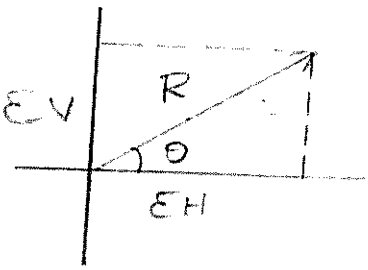
$$\Sigma H = 200\cos 26.56 + 150\cos 45^\circ - 70\cos 50^\circ - 100\cos 53.13^\circ$$

$$\Sigma H = \underline{179.96N}$$

$$\Sigma V = 200\sin 26.56 - 150\sin 45^\circ - 70\sin 50^\circ + 100\sin 53.13$$

$$\Sigma V = \underline{9.73N}$$

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$



$$R = \sqrt{(179.96)^2 + (9.73)^2}$$

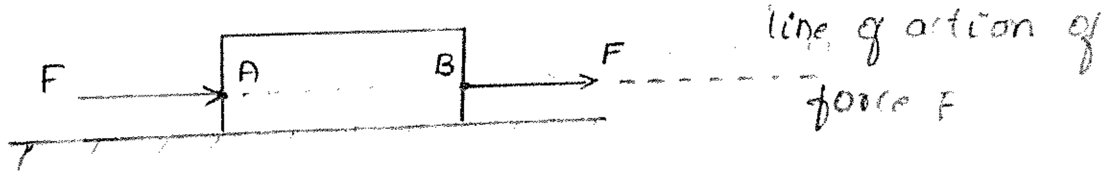
$$\boxed{R = 80.22}$$

$$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$$

$$\theta = \tan^{-1}\left(\frac{9.73}{179.96}\right)$$

$$\boxed{\theta = 3.09^\circ}$$

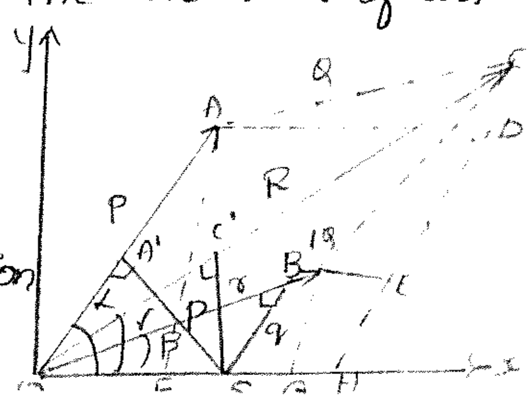
2a (i) State law of transmissibility of forces.



It states that " The point of application of a force may be transmitted (or replaced) to any point along its line of action without changing the condition of equilibrium." [i.e the effect is same].

(ii) State and prove the varignon's theorem ?

The moment of a force [i.e resultant) at any point is equal to the algebraic sum of the moments of its components about that point.



Proof :-

R is the resultant of P and Q

Let α, β and γ be the inclination of P, Q and R w.r to X-axis

Let S be the point on x-axis.

Draw SA', SB' and SC' $\perp^r$ to the lines of action of forces P, Q and R from S.

Draw AF, BG and CH $\perp^r$ to X-axis and drawn AD & BE parallel to x-axis.

From the figure,

$$CH = DH + CD$$

$$\therefore R \sin\gamma = P\sin\alpha + Q\sin\beta.$$

x'ly the above eq^n by the distance OS.

$$R(OS)\sin\gamma = P(OS)\sin\alpha + Q(OS)\sin\beta.$$

$$\therefore R(SC') = P(SA') + Q(SB')$$

$$\therefore \boxed{R.r = P.p. + Q.q}$$

$\therefore$ Moment of R about S = Alegebraic Sum of the moments of P and Q about S.